Journal of Energy
VOLUME 75 Number 3 | 2026
journal homepage: http://journalofenergy.com/

The Influence of Hysteresis Modeling on the Calculation of Magnetic Fields

Stjepan Frljić, Bojan Trkulja, Ana Drandić

***Summary* —Due to hysteresis effects, the magnetization curve of ferromagnetic material is a history-dependent, nonlinear relationship between the magnetic flux density *B* and the magnetic field *H*. Here, the *H*(*t*) waveform is calculated for a given *B*(*t*) excitation using the Preisach hysteresis model. The resulting hysteresis loop indicates the importance of more sophisticated modeling of ferromagnetic materials when the excitation deviates from a pure sine waveform and the material operates near saturation. A comparison with non-hysteretic approximations based on static magnetization curves highlights their tendency to underestimate waveform distortion and harmonic content.**

***Keywords* — Hysteresis, Preisach model, magnetization curve, higher harmonics**

I. Introduction

Accurate modeling of magnetic core behaviour remains an essential aspect of modern power engineering simulations, particularly in the analysis and design of transformers, inductors, and electromagnetic actuators. While classical linear magnetic models provide analytical simplicity, they fail to capture the nonlinear and hysteretic properties inherent to ferromagnetic materials, which strongly influence magnetizing current waveforms, harmonic distortion, and loss estimation, especially under realistic operating conditions.

In many industrial finite element and circuit-based simulations, nonlinear magnetic behaviour is represented through single-valued magnetization characteristics obtained from quasi-static measurements. Such characteristics, commonly referred to as DC magnetization curves, provide a monotonic relationship between magnetic flux density and field strength and enable efficient numerical implementation [1]. However, they neglect magnetic memory effects and energy dissipation associated with hysteresis. Consequently, models based solely on these curves cannot reproduce waveform distortion or minor-loop dynamics observed in practical devices.

To address these limitations, hysteresis models are required [2], [3]. Among the available approaches, the Preisach model remains one of the most widely used phenomenological frameworks due to its mathematical generality, physical interpretability, and compatibility with numerical simulation. It represents hysteresis as the collective response of elementary relay operators, and it is capable of reproducing major and minor loops, wiping-out properties, and memory effects without explicitly modeling domain-level physics [4].

Despite its modeling power, the Preisach model introduces increased computational complexity compared to singlevalued approaches. For engineering applications, it is therefore useful to quantify the impact of including hysteresis on predicted magnetic responses. In transformer cores, for instance, sinusoidal voltage excitation produces an approximately sinusoidal flux density, whereas the resulting magnetic field strength becomes distorted due to nonlinear and hysteretic behaviour. Understanding the extent of this distortion is important for assessing current harmonics, electromagnetic interference, and loss mechanisms.

The objective of this work is to investigate magnetic response differences between a scalar Preisach hysteresis model based on an analytical Everett function and two conventional nonlinear single-valued magnetization models corresponding to the same material properties. Emphasis is placed on waveform distortion and spectral content under realistic, minimally distorted sinusoidal flux excitation.

II. Hysteresis Model

Rate-independent hysteresis losses and eddy current losses can be treated separately, as explained in detail in [5]. Therefore, we focus here solely on hysteresis models and analyze them independently of eddy currents.

A. Scalar Preisach model

The scalar Preisach model describes hysteresis as the superposition of a finite or infinite number of hysterons, i.e. elementary two-state relay operators, each characterized by two switching thresholds α and β, where $\alpha \geq \beta$, as shown in Fig. 1. Each relay assumes one of two states depending on the history of the input magnetic field $H(t)$. The macroscopic magnetization is obtained by integrating the contributions of all hysterons weighted by a distribution function over the Preisach plane. Mathematically, the magnetic flux density can be expressed as

$$B(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} H(t) d\alpha d\beta \tag{1}$$

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Stjepan Frljić, Bojan Trkulja, Ana Drandić, The Influence of Hysteresis Modeling on the Calculation of Magnetic Fields, Journal of Energy, vol. 75 Number 3 (2026), 69–73
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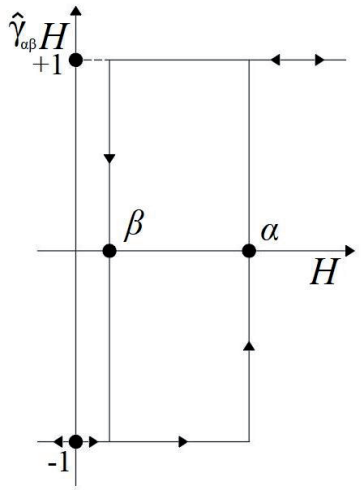


Fig. 1. Diagram of a single hysteron with threshold values α and β. The output is binary, taking only the values +1 and −1.

where $\mu\ (\alpha\ \beta)$, is a Preisach density function which is used as a weighting function, and $\hat{\gamma}_{\alpha\beta}$ is a two-state hysteron operator with the values +1 and −1 as its only two possible states (outputs). The region of hysterons with output $\gamma_{\alpha\beta}H=+1$ and the region with output $\gamma_{\alpha\beta}H=-1$ are denoted in Fig. 2 as S^+ and S^- respectively, and they are generally separated by a single polygonal (staircase line) boundary. Since hysterons are arranged in the Preisach plane according to their field threshold values α and β, it is impossible for "islands" of hysterons with output $\gamma_{\alpha\beta}H=+1$ to form within the region of hysterons with output $\gamma_{\alpha\beta}H=-1$, and vice versa.

Integration in (1) is carried out over the Preisach triangle, since $\alpha \geq \beta$ and the existence of $\alpha=H_{max}$ and $\beta=H_{max}$ are assumed in the model. When integrating the contributions of all hysterons within the Preisach triangle, they are weighted by the function μ ($\alpha\ \beta$).

Hence, the output B(t) is encoded in a moving boundary within the Preisach plane separating hysterons in positive and negative states, as shown in Fig. 2. This geometric representation naturally reproduces hysteresis memory properties such as congruency and wiping-out. However, it should be noted that the classical scalar Preisach model remains phenomenological and does not explicitly represent domain-wall physics or interaction fields.

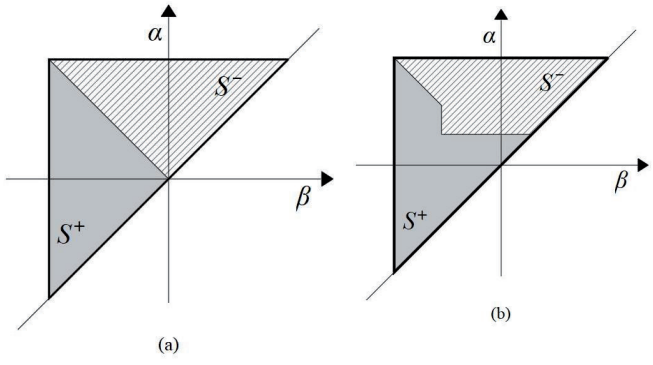


Figure 2. Preisach plane: (a) Completely demagnetized state, and (b) some general state of magnetization. The boundaries of the Preisach triangle are defined by the maximum values $\alpha = +Hmax$ and $\beta = -Hmax$ and with the hypotenuse being the line $\alpha = \beta$.

B. Everett Function Formulation

Direct numerical integration over the Preisach plane, using the Preisach density function $\mu\ (\alpha\ \beta)$, is computationally expensive. An equivalent formulation utilizes the Everett function $E(\alpha\ \beta)$, which represents the magnetization change associated with transition between the two magnetization states, more precisely

$$E(\alpha,\beta)=2\cdot\iint_{\alpha\geq\beta}\mu(\alpha,\beta)d\alpha d\beta \qquad (2)$$

The Everett function significantly reduces computational effort by transforming area integrations into evaluations of polygonal boundary between S^+ and S^- regions, especially when available in analytical, closed-form. The use of analytic Everett functions offers several advantages, including reduced memory requirements, smooth differentiability, parameter-based tuning and simplified inverse modeling.

In the present work, a continuous scalar Preisach model is used, i.e. an infinite number of hysterons is assumed, and the following Everett closed-form function is employed in the calculation:

$$E(\alpha,\beta)=\frac{c}{d+e^{-k\alpha}+e^{k\beta}}-\frac{c}{d+e^{k\alpha}+e^{-k\beta}}, \qquad (3)$$

where α and β are the corresponding ascending and descending threshold values of the H field, and c, d, and k are the material identification parameters. A surface plot of one possible Everett function (3) is shown in Fig. 3. It should be noted that the function is defined only over the Preisach triangle.

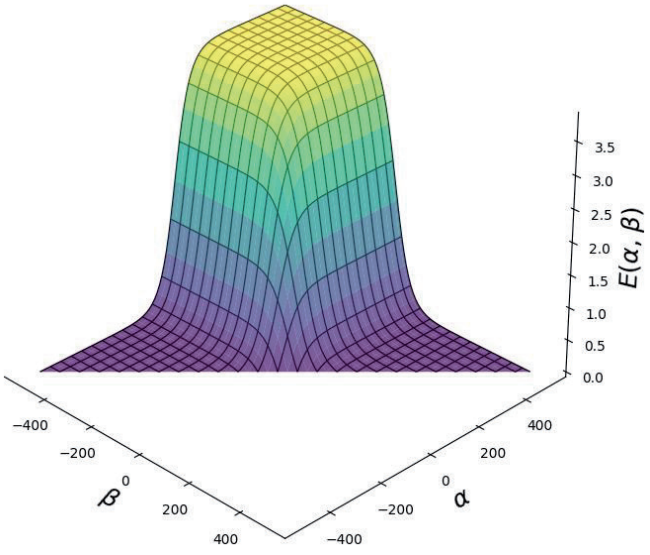


Figure 3. Everett function (3) for the values c = 1, d = 0.26, and k = 0.03

Analytical closed-form Everett expression in (3) has a small number of shape and scaling parameters. If more parameters are needed, the Everett function (3) can be augmented with additional terms as in

$$E(\alpha,\beta)=\frac{c}{d+e^{-k\alpha}+e^{k\beta}+e^{-g\alpha}+e^{g\beta}}-\frac{c}{d+e^{k\alpha}+e^{-k\beta}+e^{g\alpha}+e^{-g\beta}}. \qquad (4)$$

Such parameterizations allow for continuous representation of hysteretic behaviour, eliminating the need for numerical integration.

Stjepan Frljić, Bojan Trkulja, Ana Drandić, The Influence of Hysteresis Modeling on the Calculation of Magnetic Fields, Journal of Energy, vol. 75 Number 3 (2026), 69–73
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III. Single-Valued Non-Linear Magnetization Curve

Conventional nonlinear magnetic models are based on a single-valued relationship between flux density and magnetic field strength, i.e. $B(t) = f(H(t))$. Such models are widely employed in finite element simulations due to their numerical simplicity and stability. While computationally efficient, this representation cannot reproduce minor loops, path dependence, energy dissipation and higher harmonics linked to magnetic memory. The comparison with the scalar Preisach model therefore highlights the influence of hysteresis on predicted electromagnetic behaviour.

Here, two such non-hysteretic representations are used, both constructed using the scalar Preisach model in order to ensure correspondence between hysteretic and non-hysteretic models.

A. Virgin magnetization curve

In the classical scalar Preisach formulation the anhysteretic and virgin magnetization curves coincide, since the model lacks energy-barrier relaxation mechanisms present in real materials. Hence, the computation of the virgin magnetization curve was straightforward:

$$B = 0.5 \times E(H, -H), \tag{5}$$

where function $E(\cdot,\cdot)$ represents Everett function defined in (3).

B. Mean magnetization curve

In practical engineering applications, magnetic materials are often represented by a measured DC magnetization curve, intended to approximate hysteretic effects through a single-valued nonlinear relation when predicting the response $H(t)$ to a prescribed $B(t)$. Compared to the virgin magnetization curve, the mean magnetization curve generally provides a more representative description of the effective material behaviour under sinusoidal excitation. The mean magnetization curve is obtained by averaging the ascending and descending branches of the major hysteresis loop, thereby reducing path dependence and capturing the central trend of the magnetization process. Although this approximation neglects memory and energy dissipation mechanisms, it offers a numerically stable and computationally efficient surrogate commonly used in transformer and finite-element simulations where full hysteresis modeling is impractical.

IV. Numerical Example

In the following, a comparison between the scalar Preisach model and a nonlinear non-hysteretic model is presented. A homogeneous magnetic field distribution within the material is assumed, thus omitting any explicit reference to the geometry of the ferromagnetic material. Eddy currents are excluded from the analysis to maintain focus on the pure effect of hysteresis. However, the model is fully compatible with conventional eddy current weak formulations [6].

A. Forward scalar Preisach model

First, the capabilities of the scalar Preisach model are demonstrated using the forward scalar Preisach model, particularly regarding the modeling of higher harmonics and their influence on the relationship between H(t) and B(t).

The following multiharmonic waveform is used as the excitation field within the ferromagnetic material:

$$H(t) = 180\sin(2\pi t) - 35\sin(6\pi t) - 60\sin(10\pi t). \tag{6}$$

Using the forward scalar Preisach model based on (3), for the given H(t) the corresponding B(t) is calculated. Both are shown in Fig. 4, in the time domain and as a B-H curve. For the parameters in (3), the values $c = 1$, $d = 0.26$, and $k = 0.03$ are used, and the resulting Everett function is shown in Fig. 3. An input field $H(t)$ with pronounced higher harmonics, i.e. high total harmonic distortion (THD), was employed to highlight the hysteresis effect, manifested as sinusoidal distortion of $B(t)$ and the minor hysteresis loops shown in Fig. 4.

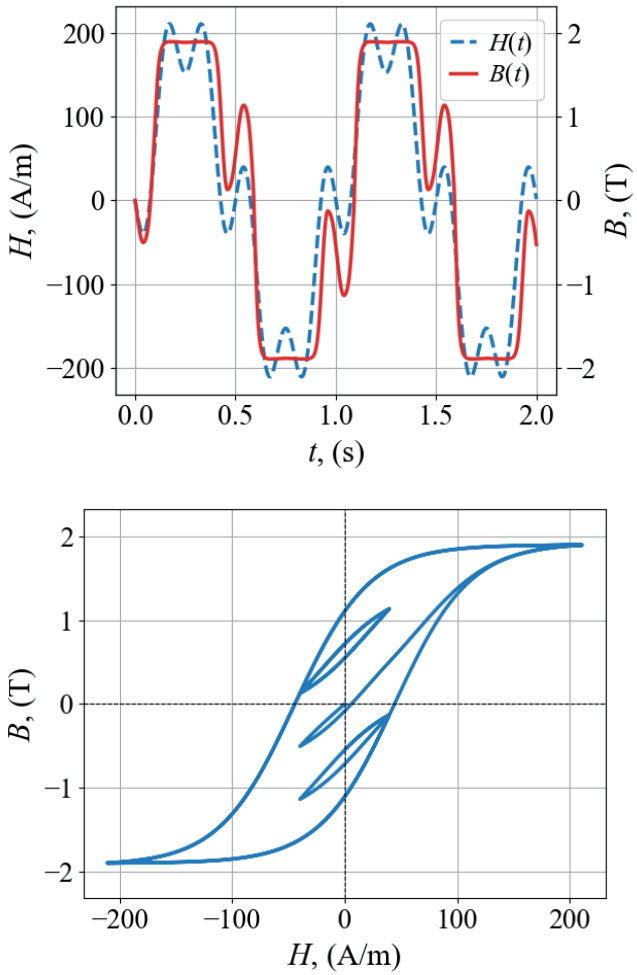


Figure 4. The time-dependent input field H(t) and resultant field B(t) (left), and the corresponding hysteretic H-B curve (right)

B. Inverse scalar Preisach model

As evident from (1), the previously presented forward scalar Preisach model produces the magnetic flux density $B(t)$ as an output for a prescribed magnetic field excitation $H(t)$. In practical power engineering applications, however, the situation is typically reversed. The flux density is directly related to the applied voltage through Faraday's law, while the magnetic field strength corresponds to the magnetizing current. Consequently, in simulation scenarios where the excitation is defined by voltage, the flux waveform $B(t)$ is known a priori, and the corresponding field response $H(t)$ must be reconstructed. This necessitates the formulation of an inverse scalar Preisach model, utilizing the forward model presented earlier. The following waveform is used as the excitation field within the ferromagnetic material:

$$B(t) = 1.8\sin(100\pi t) - 0.72\sin(500\pi t) + 0.018\sin(700\pi t)\ \mathrm{T}. \tag{7}$$

The availability of a closed-form Everett function enables

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efficient forward simulation of magnetization trajectories while preserving the fundamental properties of the Preisach framework. At the same time, the analytic structure simplifies construction of an inverse mapping because it allows repeated evaluation of the forward model without requiring numerical integration in the Preisach plane. The inverse model implemented in this work follows a numerical root-search strategy in which the magnetic field value at each time step is determined such that the forward model reproduces the target flux density.

Specifically, the algorithm reconstructs the field waveform sequentially in time. For each target value $B(t_k)$, the previously reconstructed field history is fixed, and a trial value of $H(t_k)$ is appended. The forward scalar Preisach model is then evaluated using this provisional history. The difference between the calculated and target flux density defines an objective function whose root corresponds to the correct $H(t_k)$ value. This nonlinear equation is solved using Brent's bracketing method over the admissible interval $[-H_{max}, H_{max}]$. By repeating this procedure for each time step, the algorithm reconstructs the full excitation waveform consistent with the scalar Preisach hysteresis model.

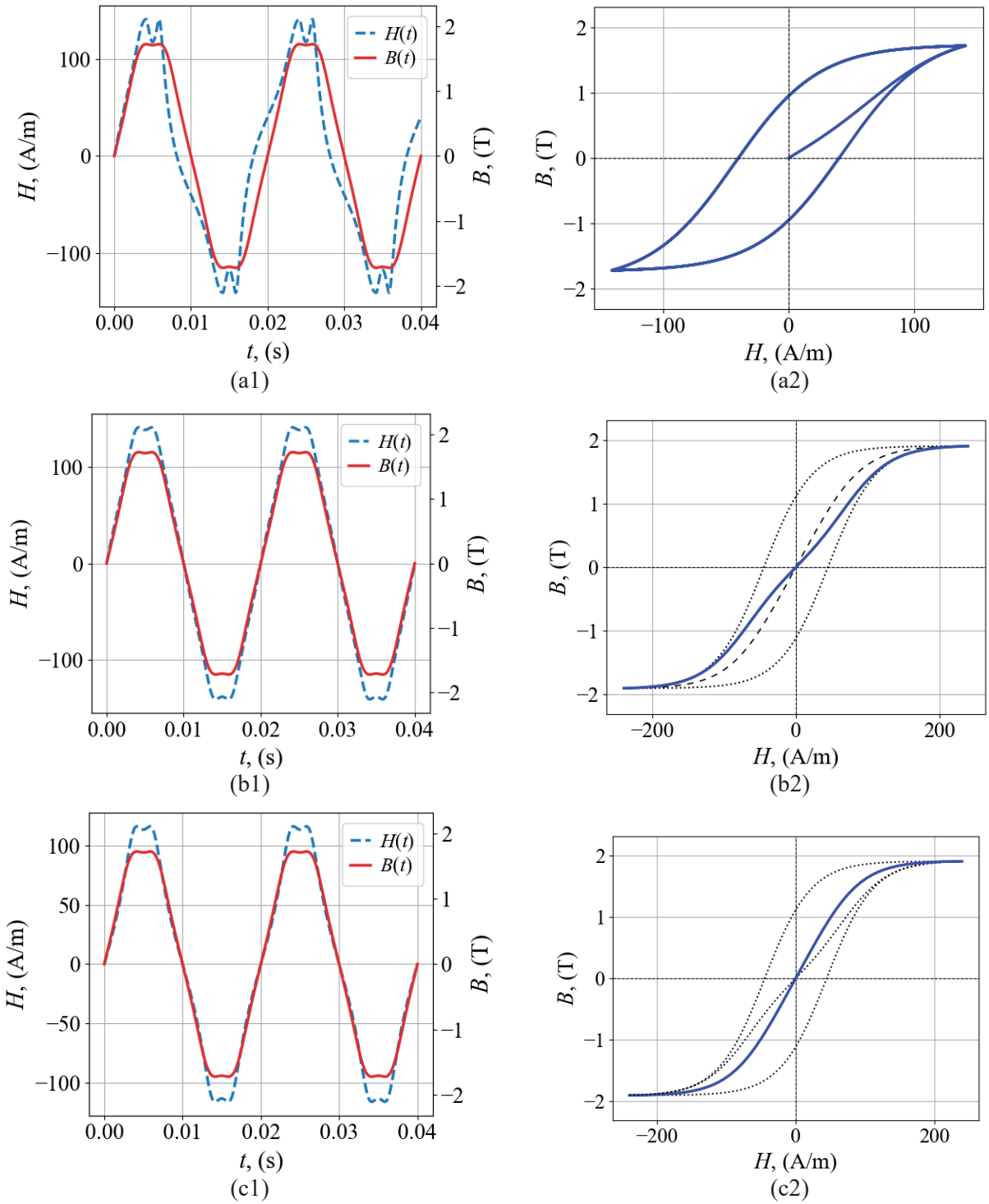


Figure 5. Comparison of simulation results for time dependence of the input function B(t) and output function H(t) on the left, and the corresponding H-B curves (solid blue lines) on the right, using: a) the inverse Preisach model; (b) virgin magnetization curve; (c) mean magnetization curve.

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Simulation results obtained using the inverse Preisach model and two non-hysteretic approximations, based on the virgin magnetization curve and the mean magnetization curve, are shown in Fig. 5. It should be noted that in Fig. 5(b2), the virgin magnetization curve is shown as a solid blue line, and the corresponding hysteresis loop (black dotted line) and the mean magnetization curve (black dashed line) are indicated for easier comparison. For the same reason, in Fig. 5(c2) the mean magnetization curve is shown as a solid blue line, together with the corresponding hysteresis loop including virgin magnetization curve (black dotted lines). A comparison of the simulation results reveals several characteristic differences in both time-domain response and spectral content. Although all three approaches are driven by the same prescribed waveform $B(t)$, the reconstructed magnetic field strength $H(t)$ differs significantly depending on how magnetic material behaviour is represented.

C. Comparison of the Preisach model and non-hysteretic models

The inverse Preisach model produces the largest deviation from sinusoidal behaviour. This is expected, since it fully accounts for path-dependent hysteretic memory and switching of elementary hysterons. The resulting *H-B* curve forms a closed loop whose width reflects energy dissipation and whose curvature varies depending on local magnetization history. These nonlinear memory effects translate into pronounced waveform distortion in $H(t)$, which is also visible in the harmonic spectrum of the SPH model in Fig. 6. The total harmonic distortion of $H(t)$ is approximately THD ≈ 21%, indicating substantial generation of higher harmonics caused by hysteresis and irreversible magnetization processes. It should be noted that the spectrum of $B(t)$ remains practically identical across all models (THD ≈ 4%), meaning that spectral differences of $H(t)$ originate solely from the material model rather than the excitation.

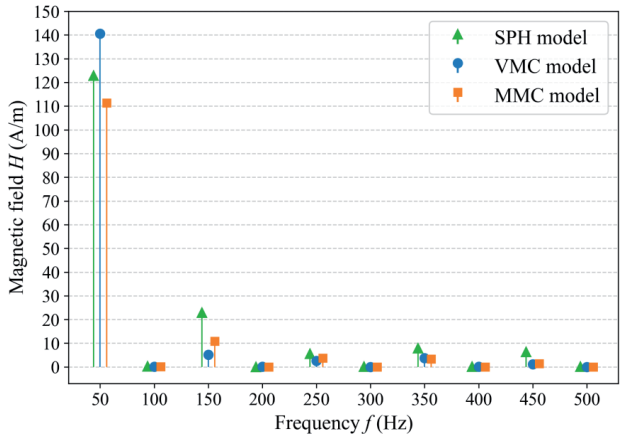


Figure 6. Harmonic spectrum of the function H(t) obtained using the inverse scalar Preisach hysteresis model (SPH model), virgin magnetization curve (VMC model) and mean magnetization curve (MMC model)

When hysteresis is neglected and the virgin magnetization curve is used as a static nonlinear mapping, the resulting $H(t)$ becomes significantly smoother. Because this curve represents a monotonic first magnetization process without loop closure, the model eliminates memory effects and effectively reduces nonlinear distortion to that associated with instantaneous nonlinearity alone. Consequently, harmonic content is strongly reduced, with total harmonic distortion of $H(t)$ falling to about THD ≈ 5%, corresponding to the harmonic spectrum of the VMC model in Fig. 6. This indicates an overly optimistic prediction of waveform linearity.

The approximation based on the mean magnetization curve provides an intermediate behaviour between these two extremes. By averaging the ascending and descending branches of the major hysteresis loop, it incorporates information about cyclic material response while still remaining memoryless. The resulting $H(t)$ waveform therefore exhibits more distortion than the virgin curve model but less than the full Preisach model, yielding total harmonic distortion of approximately THD = 11%, corresponding to the harmonic spectrum of the MMC model, shown in Fig. 6. This suggests that the mean curve more accurately captures the effective nonlinear stiffness of the material under periodic excitation than the virgin curve, although it cannot reproduce hysteretic energy dissipation or minor-loop behaviour.

Overall, the comparison highlights a clear trade-off between physical fidelity and modeling simplicity. The inverse Preisach model accurately reproduces hysteresis-induced distortion and should be preferred whenever higher harmonic generation is relevant, especially when the material is in a highly saturated regime. Non-hysteretic, nonlinear approximations significantly reduce computational cost and complexity, and the mean magnetization curve in particular offers a reasonable compromise for steady-state simulations. Nevertheless, the virgin curve approximation should be used cautiously under cyclic excitation, as it tends to underestimate nonlinear distortion and may lead to overly optimistic predictions of magnetic behaviour.

V. Conclusion

This work presents a classical scalar Preisach model based on an analytical Everett function and compares its response with conventional nonlinear single-valued (non-hysteretic) magnetization models, based on the virgin magnetization curve and mean magnetization curve. The comparison highlights the role of hysteresis in shaping magnetic field waveforms and spectral characteristics under sinusoidal magnetic flux excitation.

The presented analysis provides a foundation for evaluating when simplified magnetization models are sufficient and when hysteresis modeling becomes necessary. Future work may include incorporation of dynamic effects, or parameter identification from experimental measurements.

VI. Acknowledgement

This research was partly supported by the NextGenerationEU framework through the project ”POSEIDON” at the University of Zagreb Faculty of Electrical Engineering and Computing. The views and opinions expressed are solely those of the author(s) and do not necessarily reflect those of the European Union or the European Commission.

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