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NONLINEAR MATHEMATICAL MODEL OF HYDROELECTRIC POWER PLANT

SUMMARY

Mathematical and simulation models enable hydroelectric power unit dynamic behavior analysis using computers. In this paper, nonlinear mathematical models of all elements in hydroelectrical power plant are detailed described. Basic elements of hydroelectric power unit are water supply tunnel, surge tank, penstock, water turbine and synchronous generator. If we want more credible simulations models and calculation results, we have to use nonlinear differential equations. Using these nonlinear differential equations, a simulation model was developed for Zakucac HPP.

Key words: block-diagram model, nonlinear mathematical models, tunnel, surge tank, penstock, water turbine, synchronous generator, Zakucac HPP

1 INTRODUCTION

Hydroelectric power plant (HPP) plays an important role in the electric power system. Power from HPP is required to provide the grid regulation services. Considering that, HPP should have high availability level in all operation regimes and good dynamics behavior. The system is divided into several parts which are modeled and then simulated together. By combining individual simulation blocks of a unique simulation model of hydroelectric power plant regulation is obtained. In this paper, basic block strukture of the hydro power plant is defined (fig. 1). First, a mathematical model of water supply system (tunnel, surge tank, penstock) [5], a model of water turbine [1,3], and detailed model of a synchronous generator [2,4] are described. Then, using the obtained nonlinear differential equations, a simulation model of all elements is developed using software package MATLAB and SIMULINK. Finally, based on HPP Zakucac, simulation results were obtained and show power plant response at:

- load step disturbance of increased or decreased active power
- a case of change of reference voltage

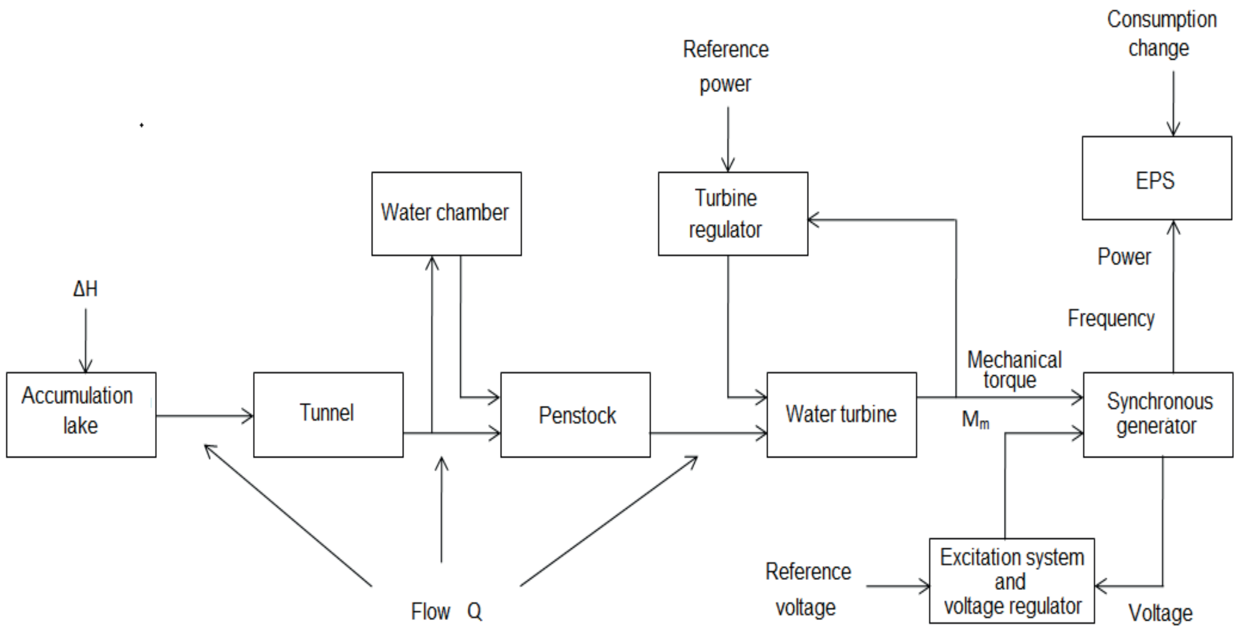

Figure 1. Basic block structure of hydro power plant

2 MATHEMATICAL MODELS OF WATER SUPPLY SYSTEM

2.1 Model of water supply tunnel

A water supply tunnel connects the accumulation lake with the surge tank. The tunnel length goes from one kilometer to several kilometers. In stationary mode, the flow through tunnel is equal to the flow through penstock. Disturbance caused by turbine power change, affects on flow changes and leads to the oscillations in a tunnel – surge tank system. Figure 2. shows the schematic representation of a water supply tunnel.

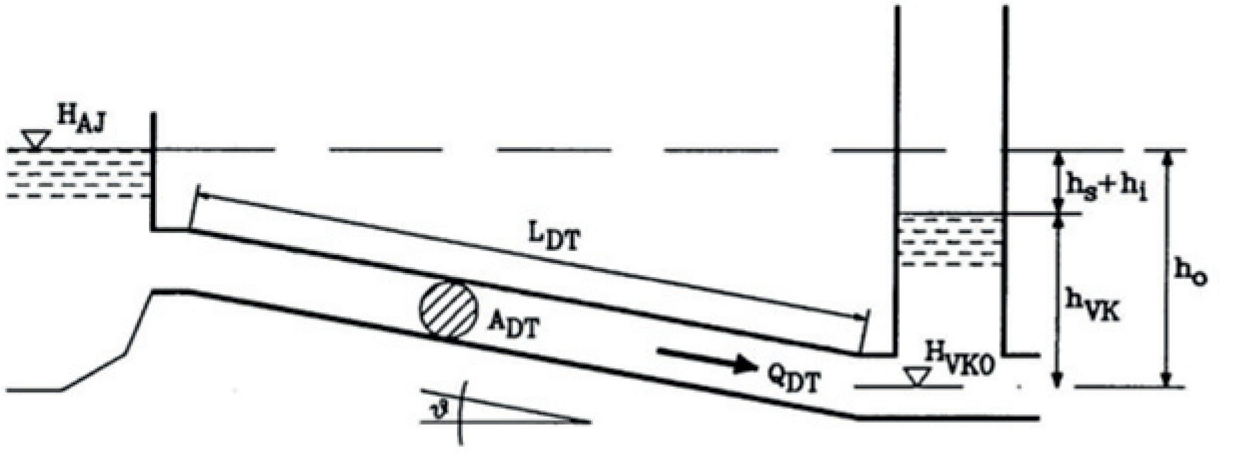

Figure 2. Schematic representation of water supply tunnel [5]

If we consider the water level of the accumulation lake constant, then the rate of change of flow in tunnel is [5]:

$$\frac{dq_{DT}}{dt} = -g\frac{A_{DT}}{L_{DT}}\frac{H_B}{Q_B}\left(h_{vk} - h_0 + k_{DT}|q_{DT}|q_{DT}\right) \qquad (1)$$

$$h_0 = \frac{H_{AJ} - H_{VK0}}{H_B} \qquad (2)$$

$$k_{DT} = \frac{1}{2}\frac{f_{DT}}{g}\frac{L_{DT}}{D_{DT}A_{DT}^2}\frac{Q_B^2}{H_B} \qquad (3)$$

$$T_{wDT} = \frac{L_{DT}\, Q_B}{g\, A_{DT}\, H_B} \qquad (4)$$

where q_{DT} is the flow through tunnel [p.u.], g is acceleration due to gravity force [m/s^2], A_{DT} is the tunnel cross section area [m^2], D_{DT} is the tube diameter [m], L_{DT} is the length of tunnel [m], H_B is rated head [m], Q_B is rated flow [m^3/s], h_{vk} is the head at surge tank [p.u.], h_0 is the static head of the water column [p.u.], T_{wDT} is the water time constant [s], k_{DT} is the factor of the head loss, f_{DT} is the friction coefficient in the conduit.

2.2 Model of the surge tank

Surge tank is an open tank which is often used with the pressure conduit of considerable length. It is required in hydro power plants for regulating the water flow during load reduction and sudden increase in the load on the hydro generator and thus reducing the pressure on the penstock. Surge tanks are generally built high enough so that the water cannot overflow even with a full load on the turbine. Figure 3. shows the surge tank with two different tanks.

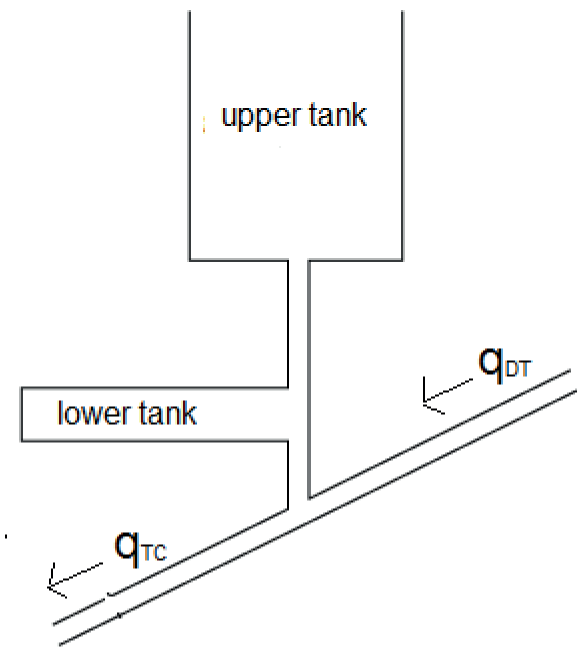

Figure 3. The surge tank

Change in the amount of water in chamber corresponds to the differences in flow through the tunnel and flow through the penstock so the rate of change of water level can be described by differential equations [5]:

$$\frac{dh_{VK}}{dt} = \frac{1}{A_{VK}(h_{VK})}\frac{Q_B}{H_B}(q_{DT} - q_{TNL}) \quad (5)$$

where $A_{VK}(h_{vk})$ [m^2] is the function of the surge tank cross section area depending on the water level h_{vk} in the surge tank.

2.3 Model of the tunnel between the surge tank and the common surge tank

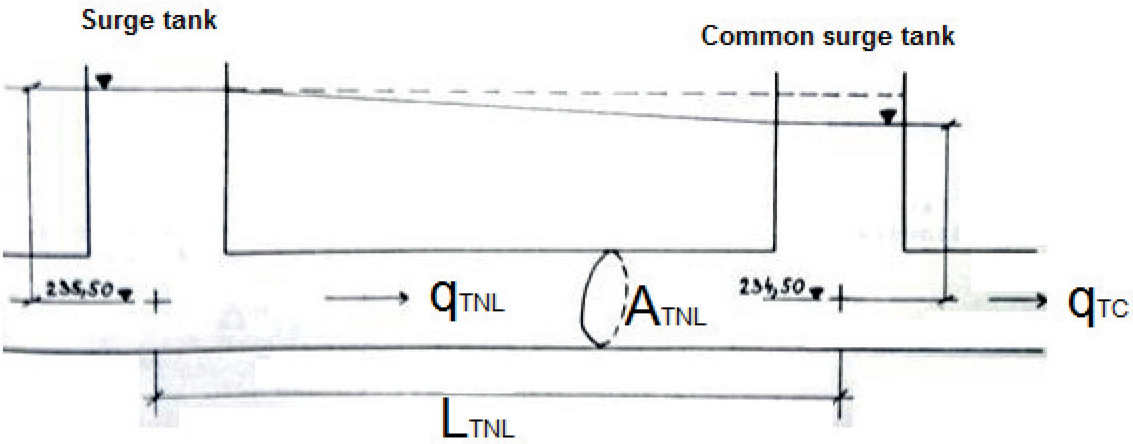

Figure 4. A schematic representation of the tunnel between the surge tank and the common surge tank

The rate of change of flow in tunnel is:

$$\frac{dq_{TNL}}{dt} = \frac{g\,A_{TNL}\,H_B}{L_{TNL}\,Q_B}\,(h_{vk} + h_{raz} - h_{zvk} - k_{TNL}|q_{TNL}|q_{TNL}) \qquad (6)$$

where q_{TNL} is the flow through tunnel [p.u.], h_{zvk} is head at the common surge tank [p.u.], h_{raz} is the difference in the geodetic height of the tunnel center at the admission to the chamber and common surge tank [p.u.]

2.4 Model of the common surge tank

This complex surge tank proved to be more economical than the classic type of the surge tank. This solution enables the use of water from both tunnels at the operation of any aggregate.

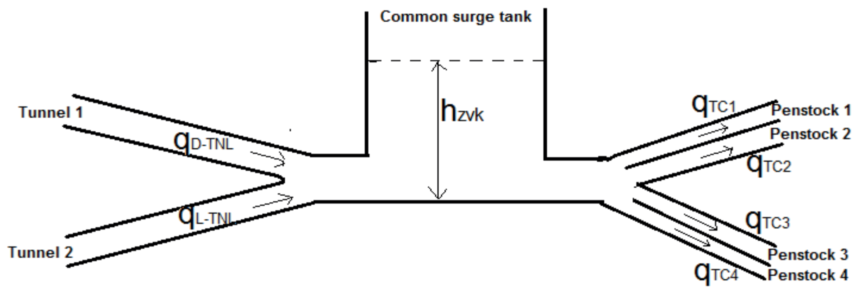

Figure 5. A schematic representation of the common surge tank

The rate of change of water level can be described by differential equation:

$$\frac{dh_{ZVK}}{dt} = \frac{1}{A_{ZVK}(h_{ZVK})}\frac{Q_B}{H_B}(q_{D-TNL} + q_{L-TNL} - q_{TC1} - q_{TC2} - q_{TC3} - q_{TC4}) \qquad (7)$$

where $q_{D\text{-}TNL}$ and $q_{L\text{-}TNL}$ are flows through right and left tunnel, respectively [p.u.], $q_{TC1...4}$ are flows through penstocks [p.u.], $A_{ZVK}(h_{zvk})$ [m²] is the function of the surge tank cross section area depending on the water level h_{zvk} in the surge tank.

2.5 Model of the penstock

Penstocks are long channels that carry water down from the hydroelectric reservoir to the turbines. Generally, they are made of steel and water under high pressure flows through the penstock. The penstock is modeled assuming an incompressible fluid and a rigid conduit.

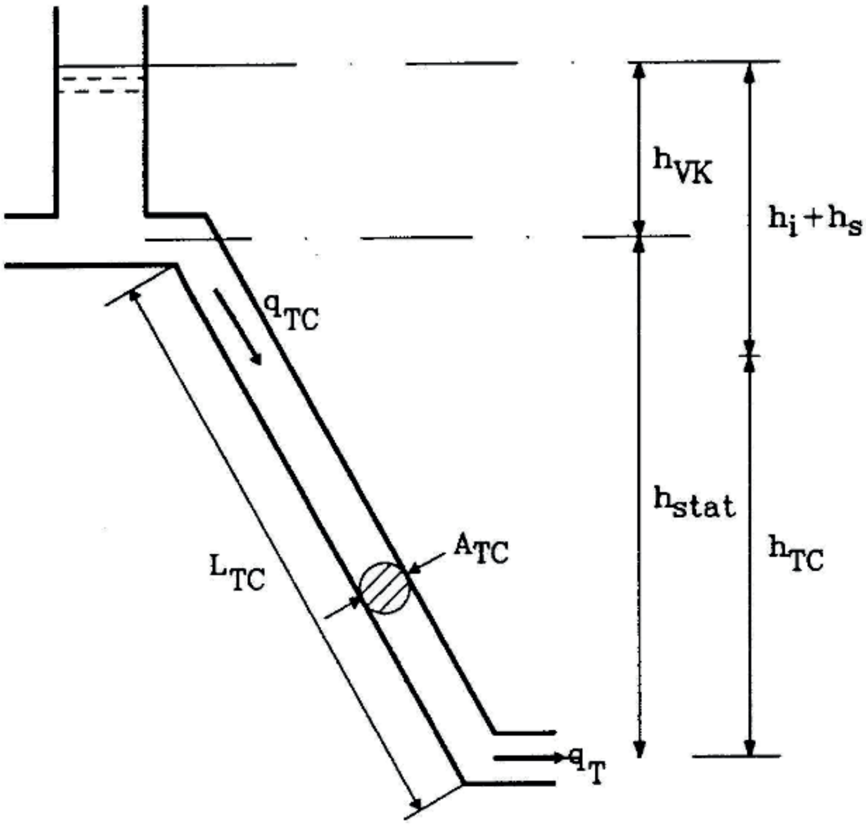

Figure 6. A schematic representation of the penstock [5]

Differential equations are [5]:

$$\frac{dq_{TC}}{dt} = \frac{1}{T_{wTC}}(h_{STAT} + h_{VK} - h_{TC}) - h_{GTC} \tag{8}$$

$$\frac{dh_{TC}}{dt} = k(q_{TC} - q_T) \tag{9}$$

$$h_{GTC} = k_{Tc} \ \frac{Q_B^2}{H_B}|q_{Tc}|q_{TC} \tag{10}$$

$$k = \frac{\pi^2}{4}\frac{a^2 Q_B}{gL_{TC}A_{Tc}H_B} \tag{11}$$

$$T_{WTC} = \frac{L_{TC}Q_B}{gA_{TC}H_B} \tag{12}$$

$$a = \frac{a_0}{\sqrt{1+\frac{E_V D_{TC}}{bE_C}}} \tag{13}$$

Where,

h_{STAT} is the static head of the water column [p.u.], h_{GTC} is the head loss due to friction in the conduit [p.u.], h_{TC} is the head at turbine admission [p.u.], q_{TC} is the flow through penstock [p.u.], q_T is the turbine flow [p.u.], L_{TC} is the length of penstock [m], A_{TC} is the penstock cross section area [m^2], k_{TC} is the loss coefficient in penstock, T_{WTC} is the water time constant [s], a_o is the speed of sound in water [1428 m/s], E_v is the modulus of elasticity of water [20.4 * 10^8 kg/ms^2], E_{TC} is the modulus od elasticity of penstock [19.6 * 10^{10} kg/ms^2 for the steel, and 19.6 * 10^9 kg/ms^2 for the concrete], b is the penstock wall thickness [m].

3 MATHEMATICAL MODEL OF WATER TURBINE

Basic elements of hydroelectric power unit as dynamic system are hydraulic turbine with conduit, governor and generator. Fig. 7 shows functional relations between elements of hydroelectric power unit with double-regulated turbine [3]:

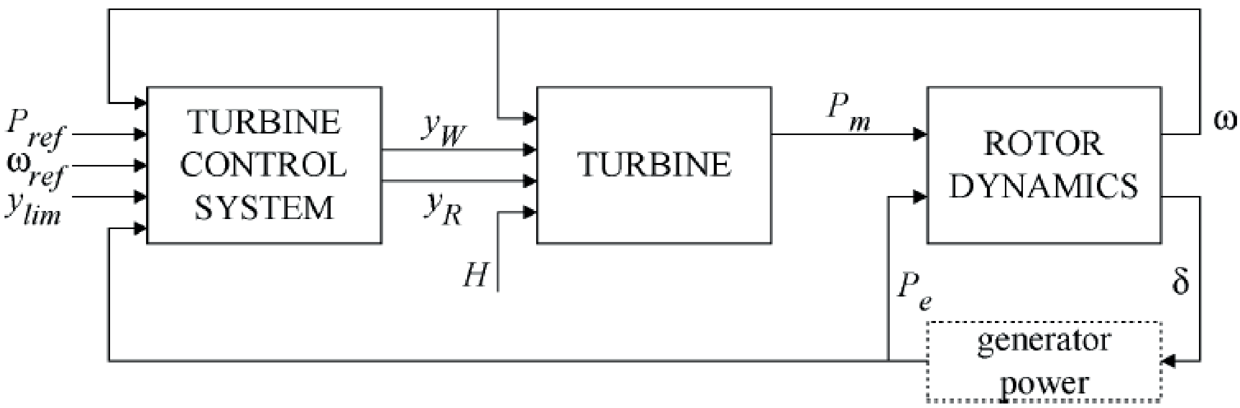

Figure 7. Structure of hydroelectric power unit with double regulated model [3]

If hydraulic turbines are installed in low head HPP with short pipelines, water and conduit can be taken as incompressible so the inelastic water column equation can be used [1,3]:

$$\frac{q_T - q_0}{h_{TC} - h_0} = -\frac{1}{s * T_{WTC}} \tag{14}$$

Where, q_T is the turbine discharge [p.u.], q_0 is the turbine discharge, initial value [p.u.], s is the Laplace operator, h_{TC} is the turbine head [p.u.], h_0 is the turbine head, initial value [p.u.].

Discharge and efficiency of double-regulated turbine are functions of head, speed, guide vanes, and runner blades opening so the mechanical power is also a function of these four variables [1,3]:

$$q = q(h, \omega, y_W, y_R) \tag{15}$$

$$\eta_t = \eta_t(h, \omega, y_W, y_R) \tag{16}$$

$$P_m = P_m(h, \omega, y_W, y_R) \tag{17}$$

In hydroelectric power unit normal operation, speed is almost constant, specially when the unit is connected to electric power system. Neglecting speed deviation, turbine discharge (15) and efficiency (16) characteristics become three variables functions.

In this paper, single regulated turbine is analysed so the assumption of turbine discharge linear dependence on wicket gate opening is usually used. In this case the curve $\boldsymbol{q(y_w)}$ is approximated with straight line through rated operating point [3].

The flow rate through turbine and mechanical power in per unit system are given by [2]:

$$q_T = y_W \sqrt{h_{TC}} \tag{18}$$

$$P_m = A_t h_{TC}(q_T - q_{NL}) \tag{19}$$

$$A_t = \frac{1}{y_{fl} - y_{nl}} \tag{20}$$

Where, y_W is per unit gate position, A_t is the turbine gain, q_{NL} is per unit no load flow, y_{fl} is the full load maximum per unit gate opening, y_{nl} is the no load per unit gate opening.

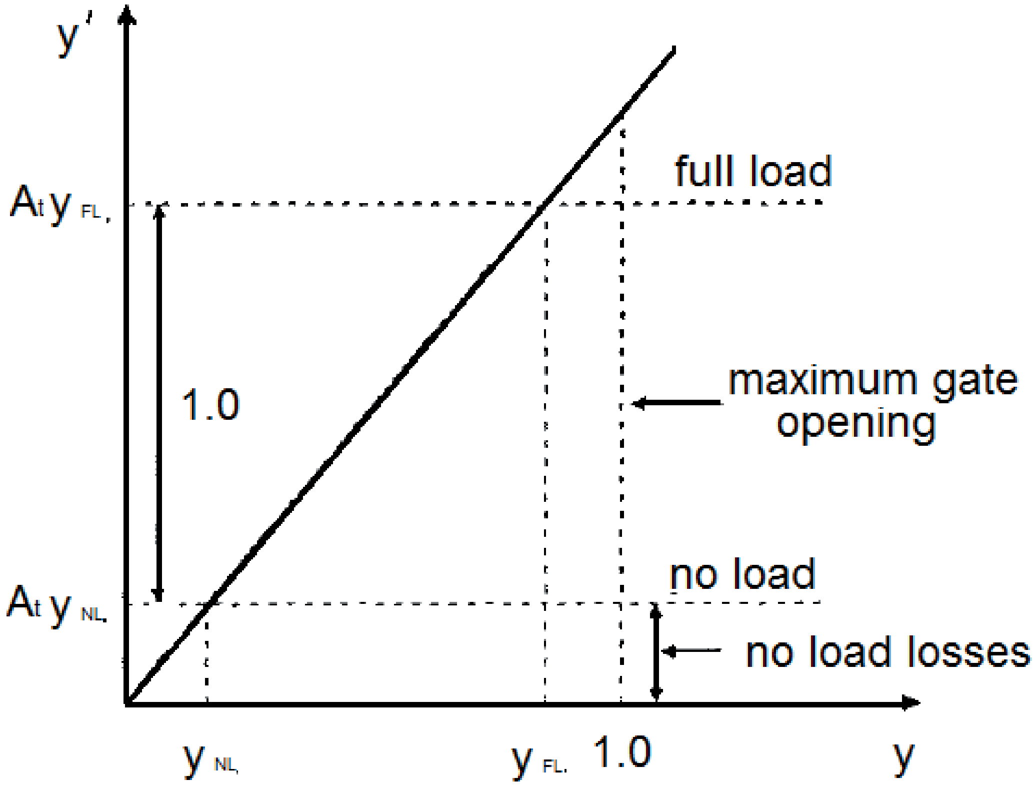

Figure 8. The relation between ideal wicket gate opening and real wicket gate opening [2]

3.1 Mathematical model of turbine governing system

Governing system or governor is the main controller of the hydraulic turbine. The governor varies the water flow through the turbine to control its speed or power output.

- <u>Speed governing [2]:</u>

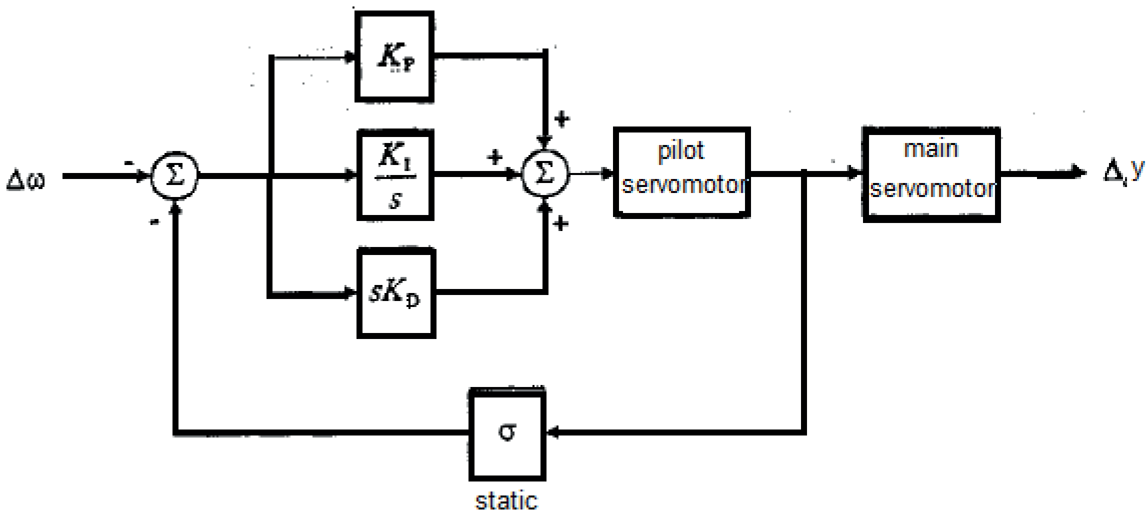

Figure 9. Block scheme of PID regulator [2]

Typical parameters values are [2] : $K_p = 3.0$; $K_i = 0.7$; $K_D = 0.5$

- Power governing:

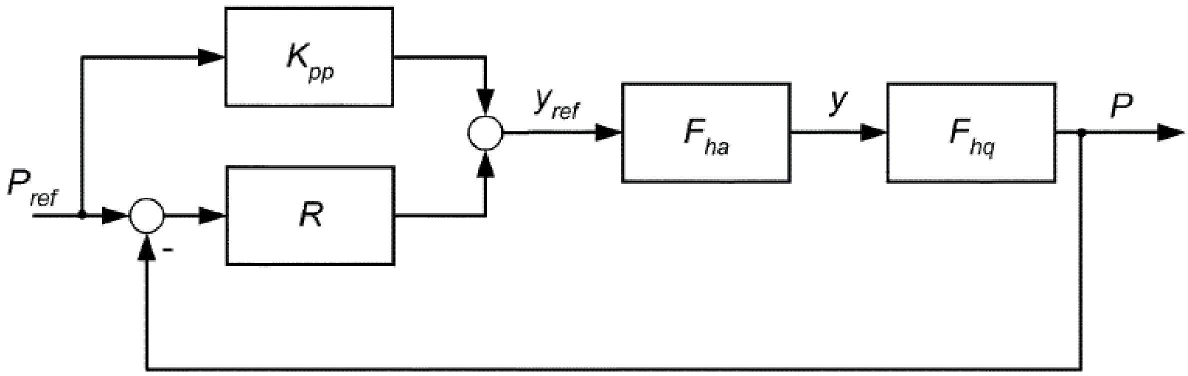

Figure 10. Block diagram of the active power control system [6]

Transfer functions of the blades positioning system and penstock system are [6]:

$$F_{ha} = \frac{y}{y_{ref}} = \frac{1}{T_a s+1} \tag{21}$$

$$F_{hq} = \frac{\Delta h_{TC}(s)}{\Delta q_{TC}(s)} = -T_W s \tag{22}$$

and a PI controller:

$$R = K_p + \frac{K_i}{s} \tag{23}$$

where T_a [s] is the time constant of the hydraulic piston positioning control system.

3.2 Rotor dynamics model

Rotor swing equation is usually written in the following form [4]:

$$J\frac{d\omega_m}{dt} = M_m - M_e \tag{24}$$

where J is the moment of inertia [kg·m^2], ω_m is the mechanical angular velocity [rad/s], M_m is the mechanical torque [Nm], M_e is the electrical torque [Nm].

Difference between mechanical and electrical torque is called an accelerating torque. Equation (24) can be written in terms of power instead of torque:

$$J\omega_m \frac{d\omega_m}{dt} = P_m - P_e \tag{25}$$

Introducing per unit values instead of real values, and with the assumption that angular velocity ω is approximately constant, the accelerating power is numerically approximately equal to the accelerating torque (p.u.), (25) becomes:

$$\frac{d\omega}{dt} = \frac{1}{2H*\omega_r}(P_m - P_e) \tag{26}$$

Where ω is the electrical angular velocity [rad/s], ω_r is the rated electrical speed [rad/s], H is an inertia constant [MWs/MVA].

4 MATHEMATICAL MODEL OF SYNCHRONOUS GENERATOR

The modelling approach presented is carried out under the following assumptions:

- Saturation effects are neglected
- Stator winding currents are assumed to set up a magnetomotive force sinusoidally distributed in space around the air gap
- The effect of space harmonics in the field distribution is neglected
- The magnetomotive force acting along the d-axis and q-axis produces a sinusoidally distributed flux wave along that axis

The most common approach is based on general two-reaction theory upon which a three-phase winding of a generator is substituted by one equivalent, fictitious two-phase winding projected onto the direct (*d*) and quadrature (*q*) rotor axis. That transformation is known as Park's or d-q transformation [4]

$$[u_{abc}] = [C_{dq0}][u_{dq0}] \qquad [i_{abc}] = [C_{dq0}][i_{dq0}] \tag{27}$$

Where the transformation matrix $[C_{dq0}]$ from the phase reference frame 'abc' to the rotor frame 'dq0' is given by [7]:

$$[C_{dq0}] = \begin{bmatrix} cos\gamma & -sin\gamma & 1 \\ \cos(\gamma-\beta) & -\sin(\gamma-\beta) & 1 \\ \cos(\gamma+\beta) & -\sin(\gamma+\beta) & 1 \end{bmatrix} \tag{28}$$

$$\left[C_{dq0}\right]^{-1} = \frac{1}{3}\begin{bmatrix} 2cos\gamma & 2\cos(\gamma-\beta) & 2\cos(\gamma+\beta) \\ 2\sin\gamma & 2\sin(\gamma-\beta) & 2\sin(\gamma+\beta) \\ 1 & 1 & 1 \end{bmatrix} \tag{29}$$

Where γ is instantaneous generator voltage angle [rad], and angle $\beta = 120^0$

Voltage equations for these linked circuits can be written:

$$-u_d = r \cdot i_d + \frac{1}{\omega_s}\frac{d\psi_d}{dt} + \omega \cdot \psi_q \tag{30}$$

$$-u_q = r \cdot i_q + \frac{1}{\omega_s}\frac{d\psi_q}{dt} - \omega \cdot \psi_d \tag{31}$$

$$u_f = r_f \cdot i_f + \frac{1}{\omega_s}\frac{d\psi_f}{dt} \tag{32}$$

$$0 = r_D \cdot i_D + \frac{1}{\omega_s}\frac{d\psi_D}{dt} \tag{33}$$

$$0 = r_Q \cdot i_Q + \frac{1}{\omega_s}\frac{d\psi_Q}{dt} \tag{34}$$

where in per units: i_d, i_q are the d and q axis generator current; i_D, i_Q are the d and q axis damper winding current; i_f is the excitation current; u_d, u_q are the d and q axis generator voltage; r is the armature winding resistance; r_f , r_D, r_Q are the field winding resistance, d and q axis damper winding resistance; ψ_d, ψ_q are the stator winding flux linkages – *0,d,q* system; ψ_f, ψ_D, ψ_Q are the field winding flux linkage, d and q axis damper winding flux linkage; ω_s is the synchronous angular frequency [rad/s].

The flux linkages equations are:

$$\psi_d = x_d \cdot i_d + x_{fd} \cdot i_f + x_{dD} \cdot i_D \tag{35}$$

$$\psi_q = x_q \cdot i_q + x_{qQ} \cdot i_Q \tag{36}$$

$$\psi_f = x_{fd} \cdot i_d + x_f \cdot i_f + x_{fD} \cdot i_D \tag{37}$$

$$\psi_D = x_{dD} \cdot i_d + x_{fD} \cdot i_f + x_D \cdot i_D \tag{38}$$

$$\psi_Q = x_{qQ} \cdot i_q + x_Q \cdot i_Q \tag{39}$$

Where in per units: x_d, x_q are the d and q axis synchronous reactance; x_f. x_D, x_Q are the field winding reactance, d and q axis damper winding reactance; x_{fD} is the field winding and damper winding mutual reactance; x_{fd} is the field winding and armature winding mutual reactance; x_{dD} is the armature winding and d axis damper winding mutual reactance; x_{qQ} is the armature winding and q axis damper winding mutual reactance.

By substituting the expressions (35-39) in (30-34):

$$\begin{bmatrix} \frac{di_d}{dt} \\ \frac{di_f}{dt} \\ \frac{di_D}{dt} \end{bmatrix} = \omega_s \begin{bmatrix} x_d & x_{fd} & x_{dD} \\ x_{fd} & x_f & x_{fD} \\ x_{dD} & x_{fD} & x_D \end{bmatrix}^{-1} \begin{bmatrix} A_d \\ B_d \\ C_d \end{bmatrix} \quad (40)$$

$$\begin{bmatrix} \frac{di_q}{dt} \\ \frac{di_Q}{dt} \end{bmatrix} = \omega_s \begin{bmatrix} x_q & x_{qQ} \\ x_{qQ} & x_Q \end{bmatrix}^{-1} \begin{bmatrix} A_q \\ B_q \end{bmatrix} \quad (41)$$

Where are:

$$A_d = -u_d - \omega \cdot \psi_q - r \cdot i_d \quad (42)$$

$$B_d = u_f - r_f \cdot i_f \quad (43)$$

$$C_d = -r_D \cdot i_D \quad (44)$$

$$A_q = -u_q + \omega \cdot \psi_d - r \cdot i_q \quad (45)$$

$$B_q = -r_Q \cdot i_Q \quad (46)$$

Inverse matrix from (40) and (41) are, respectively:

$$A^{-1} = \frac{1}{\det[A]} \begin{bmatrix} x_f x_D - x_{fD}^2 & x_{dD} x_{fD} - x_{fd} x_D & x_{fd} x_{fD} - x_{dD} x_f \\ x_{dD} x_{fD} - x_{fd} x_D & x_d x_D - x_{dD}^2 & x_{dD} x_{fd} - x_d x_{fD} \\ x_{fd} x_{fD} - x_{dD} x_f & x_{dD} x_{fd} - x_d x_{fD} & x_d x_f - x_{fd}^2 \end{bmatrix} \quad (47)$$

$$\det[A] = x_d\left(x_f x_D - x_{fD}^2\right) - x_{fd}\left(x_{fd} x_D - x_{fD} x_{dD}\right) + x_{dD}\left(x_{fd} x_{fD} - x_{dD} x_f\right) \quad (48)$$

$$B^{-1} = \frac{1}{x_q x_Q - x_{qQ}^2} \begin{bmatrix} x_Q & -x_{qQ} \\ -x_{qQ} & x_q \end{bmatrix} \tag{49}$$

Standard generator parameters are reactances as seen from generator terminals associated with fundamental frequency during steady-state, transient and subtransient states along with corresponding time constants that determine the currents and voltages falloff gradient. Hence, it is necessary to determing the other model parameters from these standard generator parameters:

$$x_{fd} = x_{fD} = x_{dD} \tag{50}$$

$$x_{fd} = x_d - x_l \tag{51}$$

$$x_f = \frac{(x_d - x_l)^2}{x_d - x_d{}'} \tag{52}$$

$$x_D = x_{fd} + \frac{(x_d{}' - x_l)(x_d{}'' - x_l)}{x_d{}' - x_d{}''} \tag{53}$$

$$x_{qQ} = x_q - x_l \tag{54}$$

$$x_Q = \frac{(x_q - x_l)^2}{x_q - x_q{}''} \tag{55}$$

$$r_f = \frac{x_f}{T_{d0}\omega_s} \tag{56}$$

$$r_D = \frac{(x_d{}' - x_l)^2}{x_d{}' - x_d{}''} \cdot \frac{x_d{}''}{x_d{}'} \cdot \frac{1}{T_d{}''\omega_s} \tag{57}$$

$$r_Q = \frac{(x_q - x_l)^2}{x_q - x_q{}''} \cdot \frac{x_q{}''}{x_q} \cdot \frac{1}{T_q{}''\omega_s} \tag{58}$$

Where $x_d{}'$ is the d-axis transient reactance in per unit; $x_d{}''$ is the d-axis subtransient reactance in per unit; $x_q{}''$ is the q-axis subtransient reactance in per unit; x_l is the stator leakage reactance in per unit; $T_d{}''$ is the d-axis open-circuit subtransient time constant [s]; $T_q{}''$ is the q-axis open-circuit subtransient time constant [s]

4.1 Load model

There are different ways to represent the load: constant impedance, constant power, constant current or any of the possible combinations of these three. For generator modelling, the load representation that will define relations between voltages, currents and angular velocity (load angle) obtained by solving the load flow is required. To simplify the generator model analysis, the rest of the electric power system is replaced by an infinite bus, thus the system influence is reduced to an impedance, and magnitude and angle of the voltage phasor at the infinite bus [4].

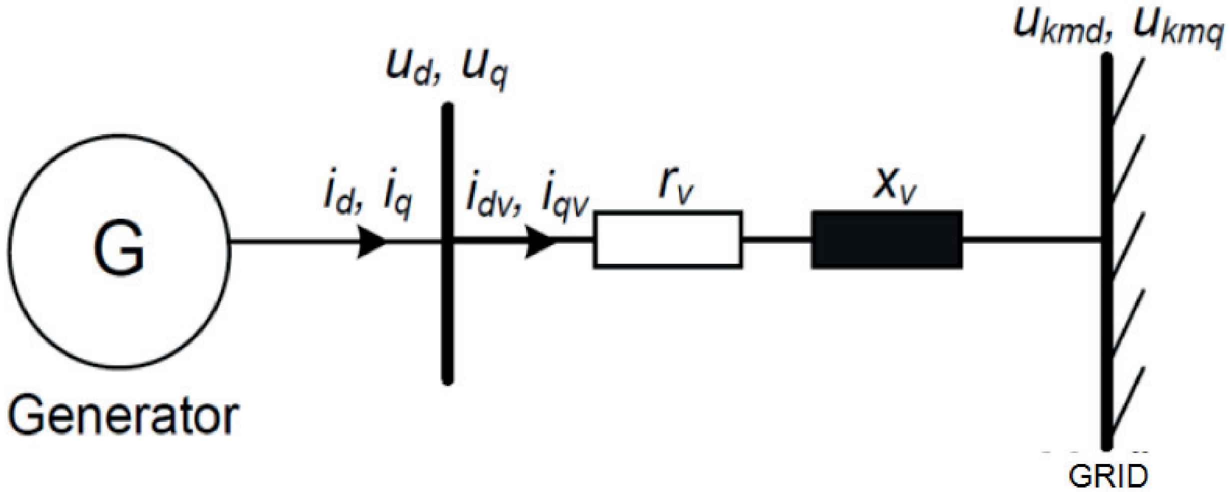

Figure 11. Generator grid connection

$$u_d = i_{dv} \cdot r_v + \frac{x_v}{\omega_s}\frac{di_{dv}}{dt} + \omega \cdot x_v \cdot i_{qv} + u_{kmd} \tag{59}$$

$$u_q = i_{qv} \cdot r_v + \frac{x_v}{\omega_s}\frac{di_{qv}}{dt} + \omega \cdot x_v \cdot i_{dv} + u_{kmq} \tag{60}$$

$$u_{kmd} = u_{km} \cdot sin\delta \tag{61}$$

$$u_{kmq} = u_{km} \cdot cos\delta \tag{62}$$

Where u_{kmd} is the grid voltage in d-axis [p.u.]; u_{kmq} is the grid voltage in q-axis [p.u.]; δ is the generator load angle [rad]

5 HPP ZAKUCAC PARAMETERS

BASE VALUES
$H_B = 250.4\ m$
$Q_B = 220\ m^3/s$
$S_B = 540\ MVA$
$U_B = 16\ kV$

Right water supply tunnel	Left water supply tunnel
$h_0 = 0.135\ p.u.$	$h_0 = 0.135\ p.u.$
$L_{DT} = 9876\ m$	$L_{DT} = 9894\ m$
$D_{DT} = 6.1\ m$	$D_{DT} = 6.5\ m$
$A_{DT} = 29.22\ m^2$	$A_{DT} = 33.18\ m^2$
$k_{DT} = 0.245\ p.u.$	$k_{DT} = 0.171\ p.u.$
$T_{wDT} = 30.27\ s$	$T_{wDT} = 26.70\ s$
Right and left surge tank	
h_{vk} [m]	**$A_{vk}(h_{vk})$ [m²]**
3.05 - 5.45	7.06
5.45 - 12.75	$51.51 + 60.3\sqrt{3.65^{2} - (h_{vk} - 9.10)^2}$
12.75 – 15.5	51.51
15.5 - 33	*right* 28.27 and *left* 38.48
33 – 33.5	233.85
33.5 – 35.9	$\frac{11000}{2.4} \cdot (h_{vk} - 33.5) + 2000$
35.9 – 36.5	$\frac{4000}{0.6} \cdot (h_{vk} - 35.9) + 13000$
36.5 – 41.5	$\frac{8700}{5} \cdot (h_{vk} - 36.5) + 17000$
> 41.5	25700
Right tunnel	**Left tunnel**
$L_{TNL} = 220\ m$	$L_{TNL} = 220\ m$
$A_{TNL} = 29.22\ m^2$	$A_{TNL} = 33.18\ m^2$
$h_{raz} = 0.004\ p.u.$	$h_{raz} = 0.004\ p.u.$
$k_{DT} = 0.018\ p.u.$	$k_{DT} = 0.032\ p.u.$
Common surge tank	
h_{zvk} [m]	**$A_{zvk}(h_{zvk})$ [m²]**
1 - 7	7.07
7 – 14.2	$133 + 162\sqrt{3.6^2 - (h_{zvk} - 10.6)^2}$
14.2 – 43.5	133
43.5 - 47	$\frac{[9 - \frac{4}{3.5}(h_{zvk} - 47)]^2 \cdot \pi}{4}$

Right penstock 1,2	Left penstock 3,4
$h_{stat} = 0.942\ p.u.$	$h_{stat} = 0.942\ p.u.$
$L_{TC} = 366.83\ m$	$L_{TC} = 366.83\ m$
$A_{TC} = 10.25\ m^2$	$A_{TC} = 12.22\ m^2$
$T_{WTC} = 3.20\ s$	$T_{WTC} = 2.68\ s$
$k_{TC} = 0.16\ p.u.$	$k_{TC} = 0.16\ p.u.$

k = 58.77 s^{-1}	k = 49.29 s^{-1}
Right turbine 1,2	**Left turbine 3,4**
q_{NL} = 0.011 *p.u.*	q_{NL} = 0.013 *p.u.*
A_t = 1.264	A_t = 1.267
PID controller	**PID controller**
T_a = 0.3 *s*	T_a = 0.3 *s*
T_m = 8.22 *s*	T_m = 8.42 *s*
K_p = 1.25	K_p = 1.18
K_i = 0.17	K_i = 0.18
K_d = 2	K_d = 1.33
PI controller	**PI controller**
K_p = 0.16	K_p = 0.12
K_i = 0.11	K_i = 0.11
Generator 1,2	**Generator 3,4**
Sn = 120 *MVA*	Sn = 150 *MVA*
Un = 16 *kV*	Un = 16 *kV*
Id = 4330 *A*	Id = 5400 *A*
cos(ϕ) = 0.9	cos(ϕ) = 0.9
x_d = 0.9 *p.u.*	x_d = 0.902 *p.u.*
x_q = 0.5 *p.u.*	x_q = 0.541 *p.u.*
x_l = 0.135 *p.u.*	x_l = 0.135 *p.u.*
r_a = 0.0025 *p.u.*	r_a = 0.0025 *p.u.*
x_d' = 0.4 *p.u.*	x_d' = 0.341 *p.u.*
x_d'' = 0.22 *p.u.*	x_d'' = 0.216 *p.u.*
x_q'' = 0.198 *p.u.*	x_q'' = 0.216 *p.u.*
T_d' = 1.2 *s*	T_d' = 3.36 *s*
T_d'' = 0.028 *s*	T_d'' = 0.082 *s*
T_q'' = 0.079 *s*	T_q'' = 0.105 *s*
H = 4.11 *Ws/VA*	H = 4.21 *Ws/VA*

6 SIMULATION RESULTS

Responses of HPP Zakucac at load step disturbance of increase and decrease active power:

- A case of load step increased active power

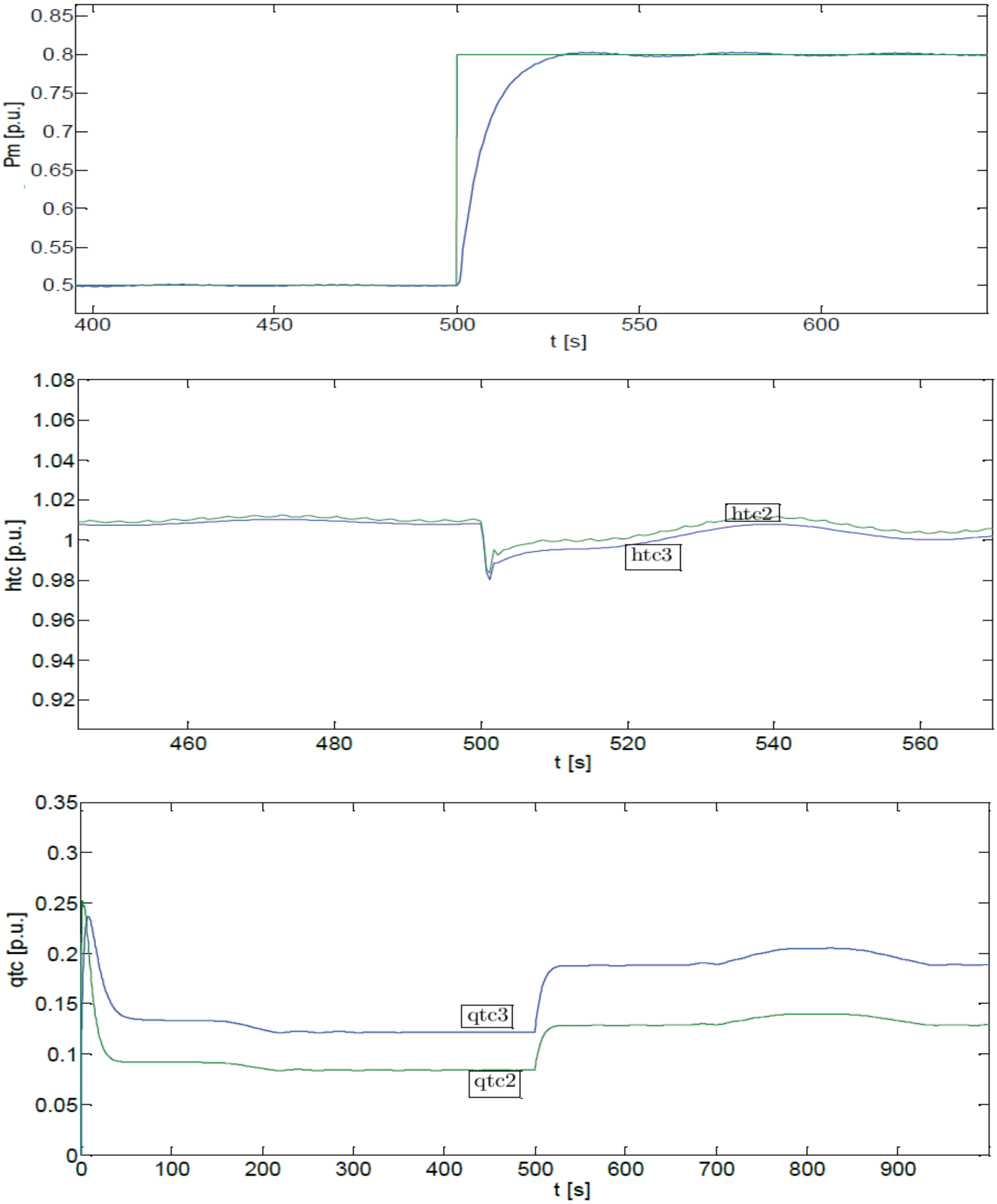

Figure 12. Responses of the power, head at turbine admission and flow through penstock

- A case of load step decreased active power:

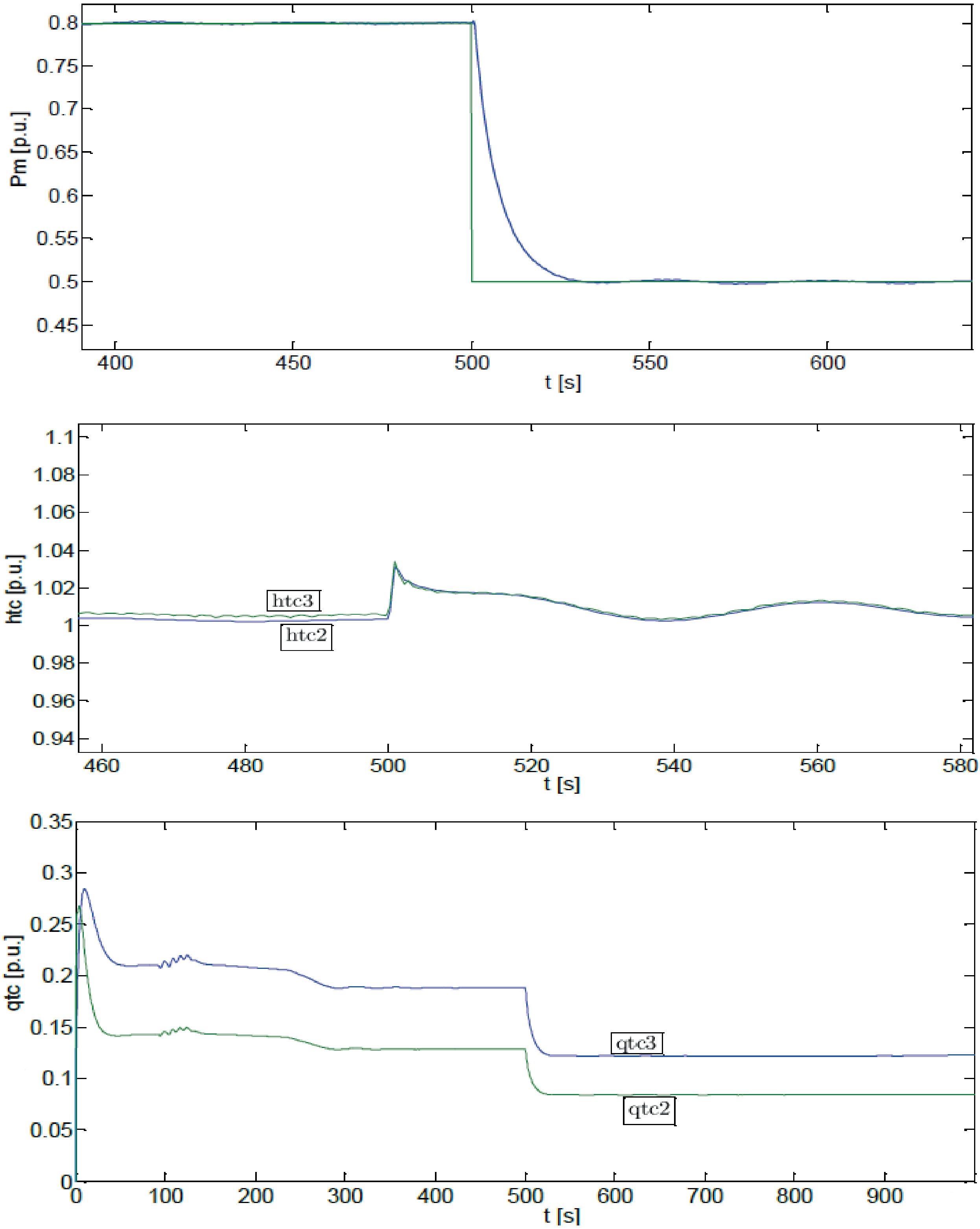

Figure 13. Responses of the power, head at turbine admission and flow through penstock

Responses of HPP Zakucac at a case of change of reference voltage:

- A case of the load step changes in voltage increase 1 – 1.05 – 1 p.u.

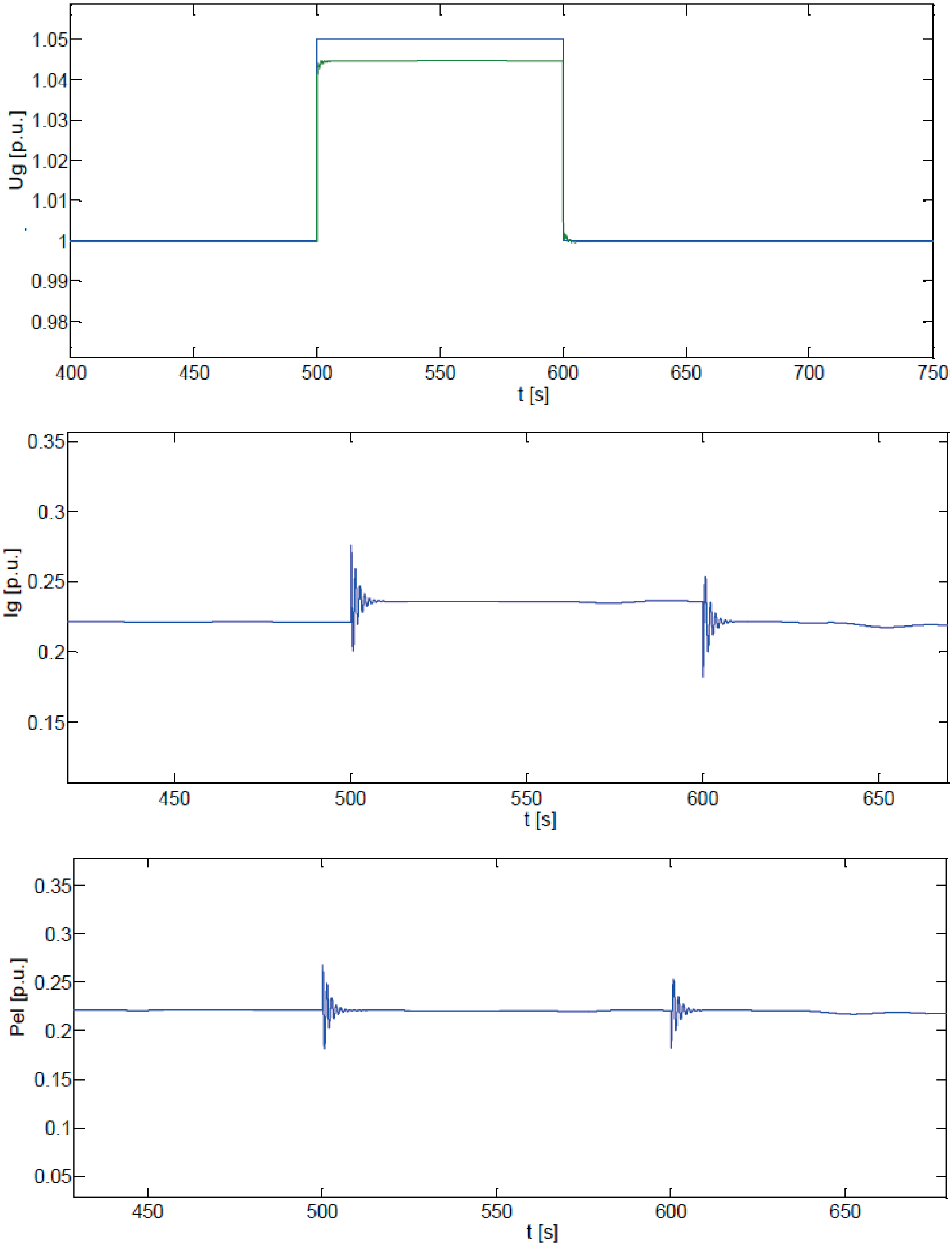

Figure 14. Responses of the voltage, current and electrical power

- A case of the load step changes in voltage decrease 1 – 0.95 – 1 p.u.

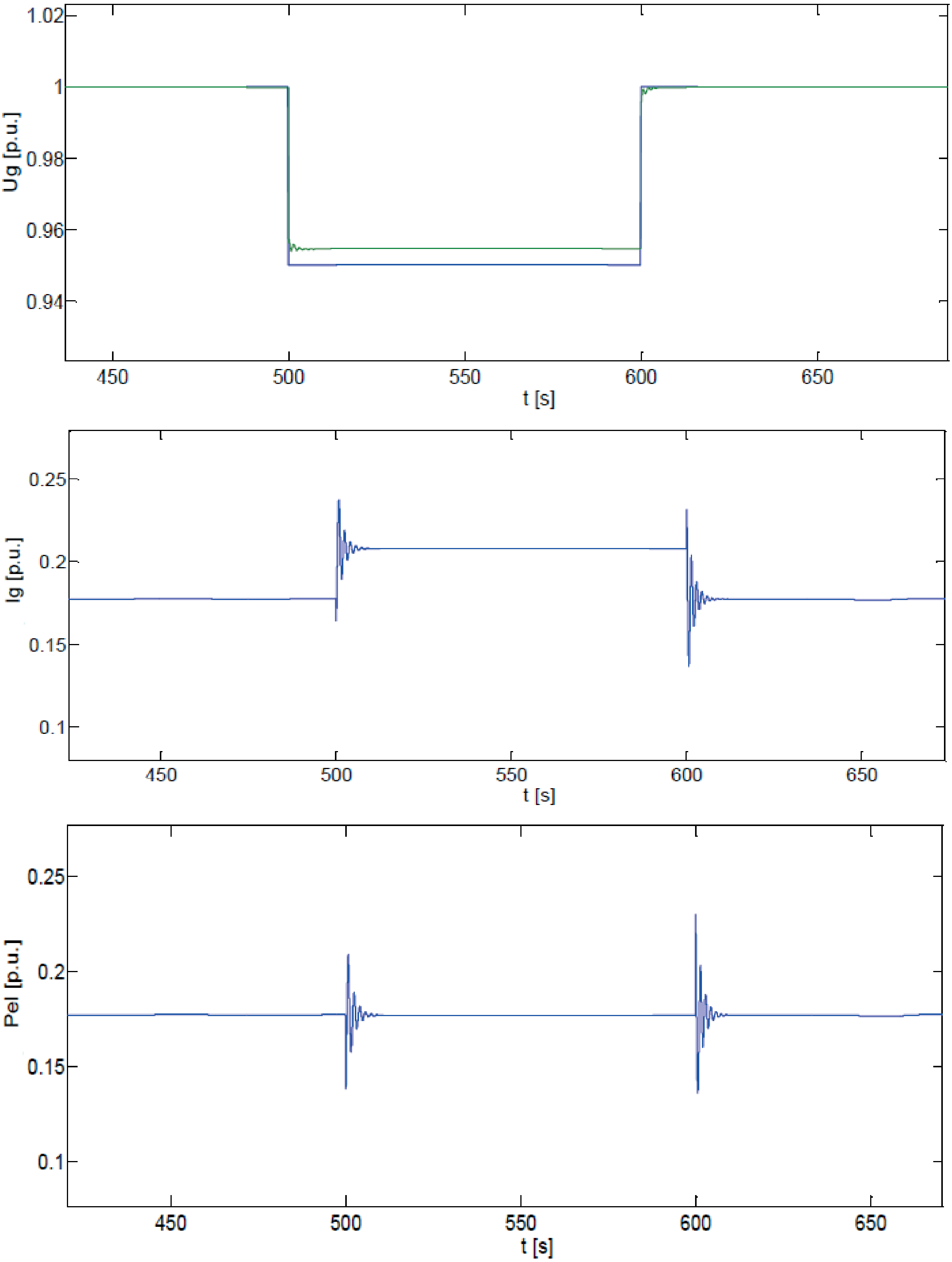

Figure 14. Responses of the voltage, current and electrical power

7. CONCLUSIONS

Hydroelectric power plant has the strongest influence on the dynamics of the electric power system so in the order to analysis and research dynamics characteristics it's necessary to have appropriate mathematical and simulation model. In some case, it's not possible to solve obtain differential equation and then certain assumptions and simplification are used. Another problem is a many data for power system calculations are usually hard to obtain. All this affects the accuracy of the model. The presented mathematical and simulation model of a hydroelectric power plant allows the analysis of all electrical and mechanical units during certain disturbances.

8. REFERENCES

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