Summary — Accurately estimating the age and condition of lithium-ion batteries (LIBs) is paramount for their safe and economically viable utilization. However, assessing the degradation of these power units proves to be challenging due to their dependence on various environmental and usage factors. In this study, we propose an efficient Particle Swarm Optimization (PSO)-based Grey Theory prediction model to determine the Remaining Useful Life (RUL) of lithium-ion batteries. The proposed model utilizes PSO to optimize the coefficients of a grey prediction model, enabling accurate forecasting of the remaining useful life of LIBs. Our results demonstrate that the presented model outperforms conventional grey prediction models in terms of both accuracy and stability. Furthermore, the proposed model offers simpler predictions compared to existing models in the literature. By introducing this promising technique, our study contributes to the precise forecasting of the RUL of lithium-ion batteries and holds potential for applications in similar domains. This research serves as a significant step towards ensuring effective management and utilization of LIBs, promoting their reliability and safety.

Keywords — Particle Swarm Optimization, Lithium-ion batteries, grey model

1 INTRODUCTION

The scarcity of fossil-fuel reserves, combined with the challenges of climate change, provides a significant motivation for the development of environmentally friendly transportation systems, sustainable energy sources, and intelligent grid networks. Successful implementation of these sectors necessitates the use of energy storage systems, which has garnered notable attention from researchers in recent times. In those fields, lithium-ion batteries are widely used. They play a significant role as one of the most important components and should be closely observed and managed. To ensure the economic feasibility of electric vehicles and the infrastructure of renewable energy systems and intelligent grids, it is imperative to have extended battery lifetimes. One of the most pressing and difficult issues is battery degradation during operation, which has become a limiting factor in a battery’s lifetime. Lithium-ion batteries have become a extensively used technology due to their efficiency in storing and providing energy [4] LIBs are a class of rechargeable battery that has become well-known in recent years, because of their extended lifespan, impressive energy density and minimal self-discharge rate. Lithium-ion batteries find widespread use across various applications, including electric vehicles, consumer electronics, and stationary storage systems [2]. The high-energy density of lithium-ion batteries (LIBs) is a key advantage, as it allows for the storage of a larger quantity of energy inside a small form factor, compared to alternative rechargeable battery technologies. This attribute renders LIBs highly suitable for applications with limited space, such as mobile phones and laptop computers. In addition, lithium-ion batteries demonstrate a prolonged cycle life, indicating their ability to withstand numerous charge and discharge cycles without substantial capacity deterioration. This characteristic guarantees the ability of LIBs to undergo repeated charging and discharging cycles, thus offering extended operational capabilities and durability, [20]. RUL is a substantial concept in the realm of lithium-ion battery technology. The term “RUL” could also be defined as the amount of time a battery can be used before it needs to be replaced [20].

Model-based methods involve the utilization of mathematical models, a collection of algebraic and empirical equations, and related parameters, which require experimentation and the analysis of large datasets [1]. Model-based approaches can construct the degradation behaviour of a battery, with either physics or regression models being employed. This extrapolation can predict battery performance. Recent research has demonstrated that physics models are more accurate for long-term predictions compared to empirical models, which often result in high errors [22].

The physics-based model for Remaining Useful Performance (RUP) [15], and RUL is predicted by integrating theories concerning reaction kinetics and electrode porosity. These theories are grounded in the physical and electrochemical processes occurring within the battery [1]. This type of model, also known as a white-box model, is a mathematical modelling technique that utilizes differential equations to mimic the behaviour of the system under consideration. These models are often able to be interpretable making them a popular choice for academic research. However, the physics-based model for RUL prediction has several drawbacks, including the need for specialized knowledge, difficulty in determining or recognizing model parameters, and high computational costs. [9]. Furthermore, measuring internal impedance at low frequencies is time-consuming and difficult. Another significant issue is that noises produced by other combined components of an online system affect the accuracy of electrochemical impedance spectroscopy (EIS) measurements as a result of the low-amplitu-
de signal required for EIS measurements. To address these issues, many researches advocate prediction based on empirical regression models [8], [14].

Data-driven models are a type of behavioural model that uses historical information to estimate the future behaviour within a system, specifically used here the Remaining Useful Life (RUL) of a battery [1]. These models are based on the premise that past performance is indicative of future performance. In opposition to models based on physical equations, empirical models are founded on experimental data that can demonstrate the connections between inputs and outputs. Empirical-driven RUL prediction techniques can be classified under two groups: statistical modelling methods, such as regression paradigm, linear polynomial paradigm, and other parametric paradigms; non-parametric techniques, such as fuzzy logic systems (FLS), machine learning (ML), etc. [13]. Figure 1 illustrates the key RUL forecasting methods for lithium-ion batteries [21].

![Fig. 1. Major RUL projection approaches [21].](image)

Recently, the Deep Neural Network (DNN) has become a favourable technology for modelling a large volume of data [1]. This is due to its multilayer network structure, which allows for multiple activation or convolution operations within a single neuron, unlike the single activation function used in traditional Artificial Neural Networks (ANNs). This feature enables DNNs to be fed with data and extracts complex features and relationships from the data.

In [10] are introduced a deep neural network model combined with an exclusion layer to avoid data over-fitting. 11,345 data points were drawn from a single battery dataset for building a model. Nevertheless, the effectiveness metric the Root Mean Square Error (RMSE) was higher than expected at 3.427 due to limitations in hyperparameter tuning and insufficiently informative data for predictions. Despite these limitations, the underlying approach holds potential. Its ability to integrate an exclusion layer for preventing overfitting is a valuable innovation and could be further explored with a more comprehensive hyperparameter search and potentially richer or augmented datasets. Future research could investigate the impact of different network architectures or feature engineering techniques on model performance.

In a separate study [11] are introduced an enhanced Long-Short-Term Memory (LSTM) method that was calibrated using Dataset collected from 28 batteries (NASA, Batteries No 5–7, Battery No 18, Batteries No 45–48, Batteries No 53–56, Batteries No 29–34, Battery No 36, Batteries No 38–44, Battery No 49, Battery No 51). The efficacy of the neural network was assessed with different battery discharge variables, and they found that RMSE decreases when the neural network is trained with more histories data. Although the proposed neural network was complex in nature, it could be reduced using more efficient optimization method and model selection. However, it may not be feasible to use these methods for forecasting Remaining Useful Life (RUL) in on-site engineering applications. In this context, while the enhanced LSTM method shows promise for battery RUL prediction, its complexity might make it less suitable for direct implementation in on-site engineering applications. Simpler models or cloud-based solutions might be more practical alternatives in such settings.

The work in [24] provides an in-depth look at the conducting of the optimized Grey model GM(1, 1) for estimating the RUL of lithium-ion batteries. The authors provide a comprehensive overview of the model and its implementation to battery life prognosis. They also discuss how the model can be used to optimize battery function and extend its life. The authors provide detailed analysis of their results, which demonstrate that the model is effective in predicting remaining useful life. Overall, this article provides a valuable resource for researchers interested in using this model for battery life prediction. To recap, the article’s pros lie in its in-depth exploration of the GM(1,1) model, its practical implementation guidance, and its focus on optimizing battery life. This combination makes it a valuable resource for researchers and practitioners alike, contributing to the advancement of lithium-ion battery technology.

A solution is needed for situations where there is a limited amount of memory and computing power, which can use raw sensor data to calculate the state of health without requiring extensive preprocessing. The Grey model GM(1, N) (Tien, 2012) can be used as a modelling approach for prognosticating the Remaining Useful Life (RUL) of a lithium-ion battery. However, due to the non-linearity of the problem, traditional calibration methods such as the least square method may not provide an optimal solution. To address this issue, this work proposes a meta-heuristic method based on the behavior of natural swarms, namely the particle swarm optimization algorithm [7], to calibrate the GM(1, N) coefficients [19]. This approach involves including a convolution integral in order to correct the modeling values by GMC(1, N) model. The proposed model is then used to prognosticcate RUL of a lithium-ion battery.

In the area of battery health diagnostics, the Particle Swarm Optimization (PSO)-based Grey Prediction Model could present a compelling alternative to data-hungry deep learning approaches. Its core strength lies in marrying the simplicity and interpretability of Grey models with the accuracy-enhancing power of PSO. This could lead to a model particularly well-suited for resource-constrained scenarios, where limited data or computational power restricts the deployment of complex deep learning architectures.

II. METHODOLOGY

A. GREY MODELLING

The Grey System approach, first founded in [5], [6], is a method dedicated to resolving problems with small sample sizes and incomplete information. This method can be utilized to analyse and evaluate vague systems when a certain component of knowledge is available. It generates, excavates, and extracts beneficial information from accessible data to accurately map out system behaviours as well as their implicit laws of development. The GM (1, N) model is one of the most extensively utilized in the recent literature in...
order to express difficult behaviour through the use of a Black-
Grey-White approach [19]. This methodology offers a broad and
dynamic investigation on the relationship between parameters of a
system. The term Accumulated Generating Operation (AGO) is a
key aspect of Grey system theory, which can increase linearity and
reduce randomness in samples. The primary chart of grey foreca-
standing models presented in Figure 2. In [18] and [19] are presented
an enhanced Grey model based on the above-mentioned GM(1,
N) model, whereby modelling values are improved by including a
convolution integral GMC (1, N).

![Diagram](image)

**Fig. 2. Fundamental illustration of grey prediction frameworks [23].**

The Grey theory models have conventionally been calibrated
using the conventional least square’s approaches. However, since
the problem is nonlinear in nature, a strictly least square’s solu-

tion may not suffice to adequately address this issue. To avoid the very
long trial-and-error process, PSO can be utilized to enhance the
Grey models’ performance. The following section will review PSO
and then describe the GMC (1, N) learning algorithm’s main steps
when used in conjunction with PSO.

**TRAINING GMC (1, N) BY PSO**

In [7] are proposed Particle Swarm Optimization (PSO) as a
different evolutionary technique to existing algorithms. PSO draws
upon the behaviour of real swarms, such as fish schools and bird
flocks, and utilizes simple structures with a clear physical meaning
for its optimization methodology. The algorithm forms a popula-
tion of individuals-known as particles-where each behaves like an
individual solution to the model, represented in an N-dimensional
space. Each particle adjusts its location within this space using its
own experience and the experience of its neighbours with regard
to their current positions, velocities, and best previous positions.
Unlike traditional algorithms that require the objective function to
be differentiable, PSO is not constrained by such assumptions
about the problem being solved. This makes it uniquely suitable for
optimizing Grey model parameters without relying on the standard
algorithms.

In this part, the primary steps of GMC(1, N) are depicted and its
optimization procedure using PSO discussed. As GMC(1, N) aims
to show the long-term behaviour of data and minimize the effect of
random occurrences by conducting the AGO on the raw data,
the first operation for building GMC(1, N) is to apply the initial
Accumulated Generating Operation to the raw data. To calibrate
the GMC(1, N) model, a suitable optimization technique such as
PSO algorithm is employed for its potential to enhance complex
numerical functions. Subsequently, an Inverse Accumulated Gene-
rating Operation (IAGO) is employed for the prediction of Remai-
ning Useful Life (RUL). Figure 2 displays a schematic diagram of
PSO-based Grey model detailing its modelling process. The mo-
delling process is outlined in the following section:

**Step #1:** Examine the raw RUL data series as:

\[ X^{(1)}_1 = \{x^{(1)}_1(1 + r), x^{(1)}_1(2 + r), \ldots, x^{(1)}_1(n + r)\}, \]

and

\[ X^{(0)}_1 = \{x^{(0)}_1(1), x^{(0)}_1(2), \ldots, x^{(0)}_1(p), \ldots, x^{(0)}_1(p + n)\}, \]

where \( j = 2, 3, \ldots, N \), \( s \) is the delay period of the system, the raw
RUL data series has a length of \( p \), while the number of values to be
estimated is indicated by \( n \).

**Step #2:** The t-\textsuperscript{order AGO sequences were obtained by pro-
cessing the raw data of each variable (i.e., time series) using t-AGO
as follows:

\[ X^{(1)}_1 = \{x^{(1)}_1(1 + r), x^{(1)}_1(2 + r), \ldots, x^{(1)}_1(n + r)\}, \]

and

\[ X^{(0)}_1 = \{x^{(0)}_1(1), x^{(0)}_1(2), \ldots, x^{(0)}_1(p), \ldots, x^{(0)}_1(p + n)\}, \]

where \( K = 1, 2, \ldots, p + n \).

Ref. [19] provides an in-depth overview of GMC(1,N), howev-
er this work will only include the core equations.

\[
\frac{\alpha X^{(k+1)} + \alpha X^{(k)}(K)}{1} + \alpha X^{(j)}(K) + \alpha X^{(j+1)}(K) + \ldots + \alpha X^{(j+k)}(K) + \ldots
\]

\[ = \alpha X^{(1)}(K) \]  \hspace{1cm} (1)

where \( K=1,2,\ldots,p+n \), the development coefficient is \( a \), \( j=2,3,\ldots,N \), the driving coefficient is \( \alpha \), and \( \alpha \) is the control parameter of a
Grey model, respectively. Consequently, the output values can be
given as:

\[
\tilde{X}^{(k)}(K) = \alpha X^{(k)}(1) + \alpha X^{(k)}(2) + \ldots + \alpha X^{(k)}(K) + \ldots
\]

\[ + \alpha X^{(k)}(K) \]  \hspace{1cm} (2)

where \( f(c) = \sum_{c=1}^{n} a \alpha X^{(c)}(c) + u \).

To compute the parameters \( a \), \( \alpha \), and \( \alpha \) the PSO algorithm could be
applied to optimize the mathematical expression in Equation (2).
The model is then calibrated until a desirable level of perfor-
mance is achieved. The final optimal parameters are used in the
GMC(1, 1) model in order to predict the RUL value. The follow-
ing is a summary of the GMC(1, N) calibration process:

A particle in the PSO algorithm is parameters in the model that
shifts its location from one iteration to the next based on velocity
equation. Generally, if the space of search is D-dimensional, then
the present velocity and position of the \( j \)th particle can be denoted
by \( A_j = [a_0, a_1, \ldots, a_{j-1}] \) and \( V_j = [v_0, v_1, \ldots, v_p]^T \) respectively.

where \( j = 1, 2, \ldots, M \) and \( M \) is the particles number of the swarm.

Particle j is able to recall the best location it has achieved so far,
referred to as the best position locally \([Pbest] = [Pbest_1, Pbest_2, \ldots, Pbest_p]^T\).
Moreover, it can also move to the best position that the
whole swarm has obtained, known as the best position globally
\([Gbest] = [Gbest_1, Gbest_2, \ldots, Gbest_p]^T\). To start with, particle j’s ini-
itial velocity and position are determined randomly. Subsequently,
particle j changes its velocity for iteration \( k+1 \) based on the best
positions (i.e. locally and globally) in addition to its velocity from
iteration \( k \) with this Equation (3):

\[
V_j(k+1) = \omega V_j + c_1 R \left(Pbest_j(k) - B_j(k)\right) + c_2 R \left(Gbest_j(k) - B_j(k)\right) \]  \hspace{1cm} (3)

Where, \( \omega \) is the inertia weight, which is conducted to regulate the
effect of the previous velocities on the present velocity. The term
\( c_1 \) is the self-confidence, and the term \( c_2 \) is the swarm-confidence
factors. \( R \) is a random number that can change from 0 to 1. The
position of particle j in iteration \( k+1 \) can be computed using the
improved velocity as follows:

\[
A_j(k+1) = A_j(k) + V_j(k+1) \]  \hspace{1cm} (4)

The value of a particle is determined by an objective function that
computes the difference between the particle and its optimum so-
lution, as follows:

\[ \text{Objective function} \]
The fitness value, $f(A_j)$, is calculated by comparing the target output $\hat{x}^{(0)}(k)$ to the predicted output $\hat{x}^{(0)}(k)$ based on the updating of the particles (i.e., solutions).

\[ f(A_j) = \sum_{k=1}^{N} (\hat{x}^{(0)}(k) - x^{(0)}(k))^2 \]  

\[ \text{(5)} \]

Step #3: In a new iteration compute the new particle velocity and particle position using formula in Equations (3) and (4), respectively, then update the model variables in Equation (2).

Step #4: If the value of error is within the model’s requirements or a set number of epochs have been completed, the calibration of the model will be finished. If not, it will go back to Step #3.

Step #5: Set the best parameters $B_j$.

Step #6: IAGO can be utilized to acquire the anticipated values. The mathematical equation is as follows:

\[ \hat{x}^{(0)}(k + r) = \hat{x}^{(1)}(k + r) - \hat{x}^{(1)}(r - 1 + r), \text{ and } \hat{x}^{(0)}(1 + r) = \hat{x}^{(1)}(1 + r). \]  

\[ \text{(6)} \]

Fig. 3: Illustrative diagram of GMC(1,1) model.

III. EXPERIMENTAL WORK

The diminishment of a battery’s effectiveness is associated with multiple processes, and its decline follows a nonlinear manner. Consequently, battery aging data must be acquired to develop an RUL prediction model and consider its accuracy and robustness. The Prognostics Centre of Excellence at NASA Ames provides a widely-utilized battery dataset [17]. This dataset includes four types of batteries packs (#5, #6, #7, and #18). Figure 3 illustrates the decrease in battery capacity that is present in the dataset.

Fig. 4: Battery capacity decay trend that is present in the dataset.

The principle of Constant Current Constant Voltage (CC-CV) is a frequently employed technique for battery charging. In this process, the current is first kept at a constant level of 1.5 A until the voltage reaches a limit of 4.2 V. After this, the voltage remains fixed while the current gradually decreases to 20 mA, thus completing the CC-CV charging process (see Figure 4). For discharging, four batteries are typically discharged at a constant current of 2 A until their respective voltages reduce to 2.7 V, 2.5 V, 2.2 V, and 2.5 V respectively.

Fig. 5: illustrates the Constant Current-Constant Voltage process used in the NASA dataset [16]. The experiment persisted until the measured actual capacity of each battery fell below 70% of its rated 2Ah. This Aging Point Threshold (APT) denoted a substantial decline in performance, signifying the point at which further exploration yielded diminishing returns. These data points are well-established in the scientific community and have been actively incorporated in current studies [3], [12].

A. MAIN STEPS IN GMC(1,1) MODELLING

Input for GM(1,1) Time Series Model:

- Time Series Data: The primary input for the GMC(1,1) model is a univariate time series dataset.
- This dataset typically includes a sequence of observations recorded over equally spaced time intervals.

Data Preprocessing:

- Ensure that the time series data is in a suitable format for analysis.
- Handle any missing values or outliers in the data.

Grey System Modeling:

- Original Data Sequence: Transform the original time series data into a first-order accumulated series.
- Establishment of Grey Differential Equation (GDE): Develop a differential equation based on the accumulated series.
- Parameter Estimation: Estimate the model parameters, which include the development coefficient and the grey input coefficient.

GM(1,1) Model Solution:

- Solve the established grey differential equation to obtain the predicted values of the original time series.

Model Evaluation:

- Assess the performance of the GMC(1,1) model using...
appropriate metrics.

Compare the predicted values with the actual values to validate the model's accuracy.

B. Results

To optimize the GMC(1, N) coefficients, the historical dataset was split into two separate sets: one for computing the model (around 48%) and the other for testing purposes (around 52%). The Python environment was utilized for the creation and simulation of the RUL model. The designed model was organized as outlined below:

Step #1: the mathematical operation (1-AGO) is used to enhance the linearity of the raw data and minimize any randomness present in the measured samples.

Step #2: the optimization algorithm PSO is used to train the GMC(1, N) model, as was mentioned in the previous section.

Step #3: The RUL value is calculated by performing an IAGO (Inverse Accumulated Generating Operation).

For the PSO algorithm, 70 particles were used with self-confidence factor $C_1$ set to 1.4 and swarm-confidence factor $C_2$ set to 1.4. The inertia weight $\omega$ was adapted over 150 epochs, decreasing from 0.8 to 0.2. At the end of this process, the total error was satisfactory (RMSE=$1.03$).

In this section, the development of another separate Grey model was conducted using the conventional Least Squares (LS) method. Three steps were involved throughout this process, with Steps 1, 2, and 3 being similar to those presented in the above mention's section. The Grey variables of Equation 2 were then determined by applying the traditional least squares method.

After training a model, it becomes essential to validate the model to evaluate its prediction quality and the accuracy of its parameters. This will provide the designer with confidence in the model and indicate whether any revisions to the training process are necessary. Model validation is a procedure that involves several steps. The exhibitions of the frameworks utilized in this exploration were determined using Mean Absolute Percentage Error (MAPE) as follows:

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|mes_i - pre_i|}{mes_i}$$

where,

$\large{mes_i}$: Measured RUL;

$\large{pre_i}$: Predicted RUL;

$\large{mes, pre}$: Average of the measured value and predicted value, respectively; and

$n$: The number of measured data.

In this section, the optimization process was applied to the final Grey theory models. Following this, a previously unused testing dataset was utilized to evaluate the performance of the models, which had not been used during the optimization stage. The outcomes acquired for the GMCPSO(1,1) model and GMC(1,1) model are exhibited in Figure 4 and Figure 5, correspondingly.

The results of this study reveal that the Mean Squared Prediction Error (MSPE) for the PSOGMC(1,1) model was 0.019, while the MSPE for the GMC(1,1) model was 0.03811. This implies that the proposed PSOGMC(1,1) model is more precise than the GMC(1,1) model in predicting RUL. This is due to the fact that the PSOGMC(1,1) model utilizes PSO for the optimization of GMC(1,1) model parameters. The PSO algorithm is an iterative optimization method that uses a population of particles to search for optimal solutions. By using this approach, the PSOGMC(1,1) model can find better solutions than those found by traditional methods such as
LS algorithms. Additionally, it was found that the true Remaining Useful Life (RUL) of the battery was 84 charge-discharge cycles in total. The RUL prediction results achieved by GMC(1,1) and PSOGMC(1,1) models were 119 and 87 respectively; demonstrating that PSOGMC(1,1) model performed better than GMC(1,1).

This study employs a Particle Swarm Optimization (PSO)-based Grey Prediction Model. This approach offers potential advantages:

**Simplicity:** Grey models are generally simpler to construct and implement compared to complex deep learning models [3].

**Limited Data:** Grey models can often perform well with limited data, which can be advantageous in situations where large datasets are unavailable.

**Optimization:** The use of PSO to optimize the grey model’s coefficients may enhance its accuracy.

Grey theory modeling is a powerful tool for predicting the RUL of lithium-ion batteries. This approach can be employed to gauge the RUL of a battery based on its current usage and state patterns. Once a prediction of the RUL has been calculated, it can be used to take decisions about when to replace a battery. For instance, if a battery has an estimated RUL of two years but is only being used once per week, then it may not need to be replaced until after 3 years have passed. On the other hand, if a battery has an estimated RUL of one year but is being used multiple times per day, then it may need to be exchanged sooner than anticipated in order to ensure optimal functioning. However, like any approach, it also has its limitations. One limitation of this approach is the reliance on optimization techniques such as PSO. While PSO can effectively optimize the coefficients of the grey prediction model, it may require significant computational resources and time to find the optimal solution. This can be a drawback in real-time or time-sensitive applications where quick predictions of RUL are required.

**IV. CONCLUSIONS**

This study presents a novel grey modelling methodology with the objective of effectively forecasting the Remaining Useful Life (RUL) of lithium-ion batteries (LIBs). The evaluation of the proposed methodology is conducted utilizing the dataset on battery charge-discharge cycles provided by NASA. The findings indicate that the RUL prediction model possesses the capability to significantly enhance the dependability and security of energy storage systems.

The grey theory modelling approach combines grey system theory and optimization techniques to model the battery’s historical data and extract valuable information for predicting its RUL. The proposed modelling method is carefully compared with existing Grey modelling methods in terms of accuracy and computational efficiency. The experimental findings demonstrate that the suggested approach outperforms the current model in both output accuracy and computational efficiency.

Furthermore, this paper provides insights into how to further improve the accuracy of RUL prediction by incorporating additional factors such as current, voltage, temperature, etc., into the model. Further research and development are necessary to address above mentioned limitations and enhance the accuracy and reliability of RUL estimation for practical applications.

**ACKNOWLEDGMENT**

We are grateful to all those who provided their support and expertise in the creation of this article, without any financial assistance.

**REFERENCES**


© 2022 Copyright for this paper by authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0)