ANALIZA UTJECAJA DISKONTINUITETA ELEKTRIČNE VODLJIVOSTI TLA NA OTPOR RASPROSTIRANJA UZEMLJIVAČA

ANALYSING THE EFFECT OF DISCONTINUITY OF ELECTRICAL CONDUCTIVITY OF SOIL ON GROUND RESISTANCE

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U članku je osim direktne primjene metode odslikavanja prikazana i primjena složenog odslikavanja, koje se javlja u slučajevima s dvije i više granica diskontinuiteta specifične električne vodljivosti. U radu je analiziran utjecaj zanemarenog odslikavanja pri određivanju otpora rasprostiranja uzemljivača, u slučaju kada se tlo modelira kao jednoslojno. Zatim je analizirana pogreška zbog modeliranja jednoslojnog tla koje se po svojoj naravi najtočnije opisuje kao dvoslojno. Prilikom modeliranja tla korištena je tehnika jednostavnog i složenog odslikavanja. Radi jednostavnosti izlaganja i usporedivosti modela za uzemljivač je odabran obični vodoravno ukopan cilindrični vodič. Dobiveni rezultati prikazani su analitički i grafički te objašnjeni.

In addition to the direct application of the method of images the article shows the application of complex images in the cases with two or more discontinuity boundaries of the specific electrical conductivity. The article analyzes the effect of neglected images in determining ground resistance, in a single-layer soil model, and goes on to analyse the error due to modelling single-layer soil which by its characteristics is most accurately described as double-layer. In modelling the soil the technique of simple and complex images was applied. To keep the presentation simple and make the models comparable, a normal horizontally inserted cylindrical conductor was taken as ground conductor. The results obtained are shown analytically and graphically and discussed.

Ključne riječi: metoda odslikavanja, otpor rasprostiranja, složeno odslikavanje, višeslojno tlo
Key words: complex imaging, ground resistance, method of images, multi-layer soil

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1 UVOD

Proračun otpora rasprostiranja uzemljivača do pojave danas vrlo dostupnih osobnih računala (PC) obavljao se najčešće primjenom empirijskih izraza. Kako je broj mogućih geometrijskih sastavnica vodiča koji tvore uzemljivač praktično beskonačan, empirijskim izrazima pokriven je vrlo mali broj, i to uglavnom vrlo jednostavnih i najčešće korištenih geometrijskih struktura uzemljivača. Osim navedenoga takvi izrazi najčešće su izvedeni ili utvrđeni mjerenjem i pod pretpostavkom jednoslojnoga homogenog tla u koji je ukopan uzemljivač. Utjecaj višestrukih diskontinuiteta specifične električne vodljivosti tla primjenom empirijskih izraza zanemarivalo se i/ili u najboljem slučaju svodilo na primjenu korekcijskih faktora, kojima se uvažavala navedena pojava. Pritom je važno spomenuti da su višestruki diskontinuiteti specifične električne vodljivosti tla pravilo u praksi, a ne pitanje akademske točnosti rezultata proračuna otpora rasprostiranja.

Svrha je ovog članka pokazati opći postupak u proračunu otpora rasprostiranja uzemljivača pri uvažavanju višeslojnog tla primjenom metode odslikavanja. Utjecaj višeslojne strukture tla na električne prilike u tlu najlakše je uvažiti primjenom metode odslikavanja izvora polja (struje odvoda uzemljivača), a samim tim i vodiča uzemljivača na granici diskontinuiteta specifične električne vodljivosti. U članku je prikazana implementacija dobivenih rezultata za proračune otpora rasprostiranja.

Radi jednostavnosti i jasnoće izlaganja prvo je prikazan izvod izraza za proračun otpora rasprostiranja vrlo duboko vodoravno ukopanog cilindričnog vodiča u tlu, a zatim su isti izrazi prošireni kako bi se uvažio utjecaj diskontinuiteta specifične električne vodljivosti na granici tlo-zrak primjenom metode odslikavanja. Pod pojmom vrlo duboko ukopanog uzemljivača misli se na dubinu ukopavanja koja je nekoliko puta veća od najizraženije dimenzije uzemljivača. Dobiveni analitički izrazi prikazani su u obliku koji je lako implementirati u algoritme za numeričke postupke poput metode momenata (MoM) za analizu složenih uzemljivača poput mrežastoga, zvjezdastog itd. Implementacija zaključaka i izraza laka je jer su navedeni složeniji uzemljivači sastavljeni od traka ili cijevi.

Zatim je prikazana primjena metode odslikavanja u slučaju kada se uzemljivač nalazi u dvoslojnom tlu. Iako je promatrani uzemljivač obična cijev kružnoga poprečnog presjeka, izrazi prikazani u ovom radu vrijede i za češće korištene trakaste uzemljivače pravokutnoga poprečnog presjeka,

1 INTRODUCTION

Until the arrival of the presently ubiquitous personal computers, the calculation of ground resistance was mostly carried out by means of empirical expressions. The number of possible geometrical components of the conductors forming the ground conductor being practically infinite, empirical expressions covered a very small number of mostly very simple and most frequently used geometric structures of ground conductors. Moreover, such expressions were more often derived or determined by measurement under the assumption of a single-layer homogenous soil into which the ground conductor is buried. The effect of multiple discontinuities of the specific electric conductivity of the soil was neglected in empirical expressions and/or reduced to the application of correction factors to make up for this occurrence at the best. It is important to note here that multiple discontinuities of the specific electric conductivity of the soil is a matter of practical rule, not a matter of academic accuracy of the results of the calculation of ground resistance.

The purpose of this article is to demonstrate the general proceeding in the calculation of ground resistance taking into account multi-layer soil by employing the method of images. The effect of the multi-layer structure of the soil on the electrical conditions in the soil is easiest to capture by employing the method of field source images (leakage current of the ground conductor) and thus of the ground conductor on the discontinuity boundary of the specific electrical conductivity. The article further shows the implementation of the results obtained in the computation of ground resistance.

For the sake of simplicity and clarity, first the derivation of the expressions for the computation of ground resistance for a very deeply burried horizontal cylindrical conductor is shown, and then the same expressions are enlarged to take into account the effect of discontinuity of the specific electrical conductivity at the soil-air boundary by using the method of images. Very deeply burried ground conductor involves the depth several times the biggest dimension of the ground conductor. The analytical expressions obtained are presented in the form that is easy to implement in algorithms for a numerical procedure such as the method of moments (MoM) to analyse complex ground conductors such as mesh grid type, star type etc. The implementation of the conclusions and expressions is easy because of the reference to more complex ground conductors made of strips or pipes.

jer se isti modeliraju uz pomoć ekvivalentnog radijusa [1]. U svim je proračunima zanemaren utjecaj spojnih vodova na uzemljivač, jer je njihov doprinos u smanjenju otpora rasprostiranja uzemljivača zanemariv.

2 FIZIKALNI MODELI TLA

U svrhu pojednostavljenja fizikalne slike te izgradnje što jednostavnijeg matematičkog aparata kojemu je točnost unutar prethodno zadanih granica, a modeli međusobno usporedivi, u prvom scenariju pretpostavlja se da je uzemljivač duboko ukopan, na taj način se može zanemariti utjecaj diskontinuiteta specifične električne vodljivosti na granici tlo-zrak. Iako je na taj način svjesno učinjena pogreška fizikalnog modela, prema kojemu se redovito ne može zanemariti utjecaj odslikavanja vodiča, primjenom ovakvog modela može se lako donijeti sud o utjecaju različitih vrsta diskontinuiteta specifične električne vodljivosti na rezultantni otpor rasprostiranja uzemljivača. Nadalje, potpuno je svejedno što je za analizu utjecaja diskontinuiteta specifične električne vodljivosti tla odabran jednostavni cilindrični vodič, jer se utjecaj višestrukih diskontinuiteta na analogan način analizira i u slučaju složenog uzemljivača ako mu je dominantna dimenzija u ravnini s ravninama diskontinuiteta specifične električne vodljivosti tla, što će detaljnije biti pojašnjeno u sljedećem radu.

2.1 Vrlo duboko ukopan uzemljivač u jednoslojnom anizotropnom homogenom tlu

Kada je uzemljivač vrlo duboko ukopan u tlo, tj. kada je njegova najistaknutija dimenzija mnogo manja od dubine ukopavanja, tada se utjecaj diskontinuiteta specifične električne vodljivosti na granici tlo-zrak, na električne prilike u okolini uzemljivača mogu zanemariti (slika 1). U tom slučaju određivanje otpora rasprostiranja uzemljivača provodi se kao da je sav prostor oko uzemljivača tlo, a granice tlo-zrak nema. Iako su okolnosti pod kojima se taj slučaj javlja rijetke, inženjeri često primjenjuju ovu tehniku za procjenu vrijednosti otpora rasprostiranja uzemljivača, a poslije uvažavaju granicu tlo-zrak u proračunima. U ovome radu za specifičnu električnu vodljivost medija upotrebljava se oznaka κ, umjesto γ, koja prema [2] označava valnu konstantu.

This is followed by a description of the method of images when the ground conductor is buried in double-layer soil. Although the observed ground conductor is a normal pipe with circular cross-section, the expressions shown in this article also apply to the more frequently used strip ground conductors with rectangular cross-section, because the expressions are modelled by means of an equivalent radius [1]. In all the calculations the effect of the connection lines on the ground conductor is neglected, because their contribution to reducing ground resistance is negligible.

2 PHYSICAL SOIL MODELS

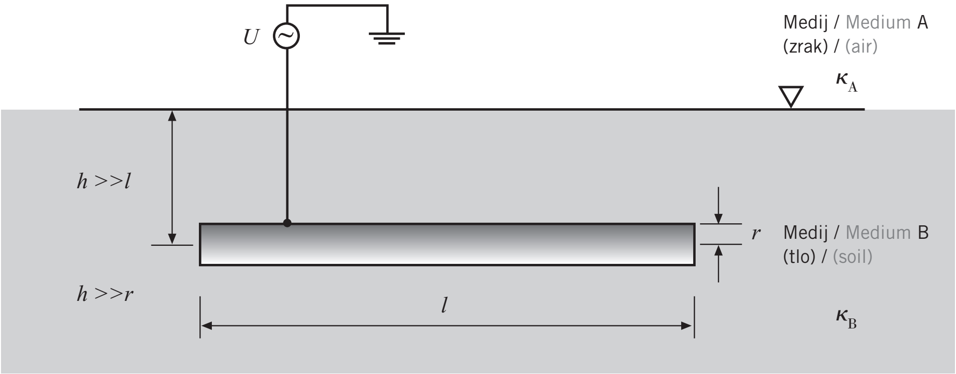
To simplify the physical picture and create the simplest possible mathematical mechanism with its accuracy within the preset limits and the models comparable with each other, under the first scenario we shall assume that the ground conductor is buried deep, so we may ignore the effect of discontinuity of the specific electrical conductivity at the soil-air boundary. Although this way an error has deliberately been made in the physical model - stating that the effect of the conductor images cannot, as a rule, be ignored - so that using such a model makes it easy to make a judgement about the effect of different types of discontinuity of the specific electrical conductivity on the resultant ground resistance. Furthermore, its is absolutely irrelevant that for the analysis of the effects of the discontinuity of the electrical conductivity of the soil a simple cylindrical conductor was chosen, because the effect of multiple discontinuities is analysed analogously in case of a complex ground conductor as well, if its dominant dimension is on the same plane with the planes of the discontinuity of the specific electrical conductivity of the soil, which will be explained in detail in the following article.

2.1 Very deeply buried ground conductor in single-layer anisotropic homogenous soil

If the ground conductor is buried very deep in the soil i.e. when its biggest dimension is much smaller than the depth to which it is buried, the effect of the discontinuity of the specific electrical conductivity on the soil-air boundary, on the electrical conditions in the surroundings of the ground counductor, can be ignored. In such a case determining ground resistance is carried out as if the entire area around the ground counductor is soil and there is no soil-air boundary. Although the circumstances under which this is the case rarely occur, engineers often use this technique to estimate the value of ground resistance, only to take the soil-air boundary into account in their later computations. In this article the specific electrical conductivity is marked κ instead of γ which according to [2] stands for wave constant.

Slika 1
Dvodimenzionalni prikaz vrlo duboko ukopanog cjevastog uzemljivača u jednoslojno anizotropno homogeno tlo

Figure 1
2D-rendering of a deeply buried pipe conductor in single-layer anisotropic homogenous soil

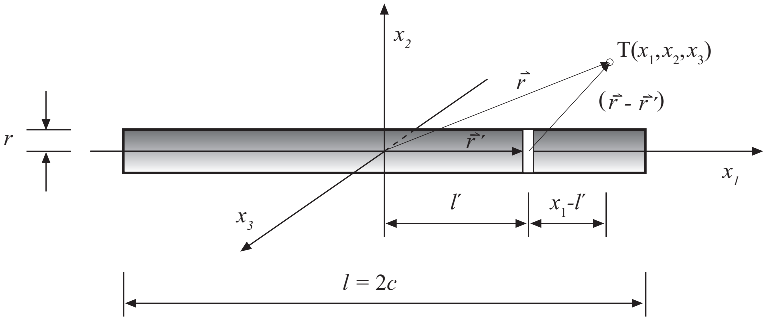


Neka je ravan vodič duljine l i radijusa (polumjera) r smješten u neograničenom prostoru specifične električne vodljivosti κ, u kojem nema nikakvih izvora polja (osim onog na promatranom vodiču). U tom slučaju riječ je o usamljenom vodiču u prostoru. Takav pristup zadaći omogućava zanemarivanje vanjskih utjecaji na električne prilike na vodiču i njegovu okolišu. Pretpostavlja se da je u vodič injektirana struja i, te da ona u cijelosti istječe iz njega (jednadžba kontinuiteta). Ishodište pravokutnog koordinatnog sustava smješteno je u središte vodiča tako da se jedinični vektor $\vec{e}_{x1}$ podudara s osi vodiča. Koordinatni sustav predstavljen je poopćenim koordinatama x_1, x_2 i x_3, kako je to prikazano slikom 2. Točka promatranja označena je s $\mathrm{T}(x_1, x_2$ i $x_3)$, a njezin položaj u odnosnu na koordinatni početak predstavljen je vektorom $\vec{r}$.

Let us assume that a straight conductor with lenght l and radius r is placed in an unlimited space with the specific electrical conductivity κ, in which there are no field sources (except the one on the conductor observed). In such a case we speak of a lone conductor in a space. Such an approach to the task makes it possible to ignore the external effects on the electrical conditions on the conductor and its surroundings. Let us assume that electricity i was injected in the conductor and that it fully flows out of it (continuity equation). The starting point for the rectangular coordinate system is placed at the center of the conductor so that the unit vector $\vec{e}_{x1}$ corresponds with the conductor axis. The coordinate system is represented by generalised coordinates x_1, x_2 and x_3, as shown in Figure 2. The point of observation is marked $\mathrm{T}(x_1, x_2$ and $x_3)$ and its position relative to the coordinate start is represented by vector $\vec{r}$.

Slika 2
Usamljeni ravni vodič smješten u neograničenom prostoru i relevantna geometrija

Figure 2
Lone straight conductor placed in unlimited space and relevant geometry



Primjenom metode superpozicije, električni potencijal u točki promatranja T(x_1, x_2 i x_3), može se odrediti kao beskonačni zbroj svih električnih potencijala koji uzrokuju točkasti izvori smješteni duž osi vodiča. Kako je veza između linijskog i točkastog izvora dana s $\Delta i=\lambda(\vec{r}')\Delta l'$, a beskonačna suma integralom, uporabom izraza za skalarni potencijal u okolišu točkastog izvora iznos potencijala u točki T(x_1, x_2 i x_3), glasi:

Using the method of superposition, the electrical potential at the observation point T(x_1, x_2 and x_3) can be determined as an infinite sum of all the electrical potentials caused by spot sources located along the conductor axis. Since the connection between a line and a spot source is determined as $\Delta i=\lambda(\vec{r}')\Delta l'$, and the infinite sum as an integral, using the expression for the scalar potential in the surroundings of the spot source the amount of the potential at the point T(x_1, x_2 and x_3) reads:

$$\varphi(\vec{r})=\frac{1}{4\pi\kappa}\int_{l'}\frac{\lambda(\vec{r}')}{R(\vec{r},\vec{r}')}\mathrm{d}l'=\frac{1}{4\pi\kappa}\int_{l'}\frac{\lambda(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}l', \tag{1}$$

gdje su: $\vec{r}'(x_1', x_2'$ i $x_3')$, $\vec{r}(x_1, x_2$ i $x_3)$ koordinate izvora polja i točke promatranja, dl put integracije, $R(\vec{r},\vec{r}')$ udaljenost između izvora i točke promatranja, a κ specifična električna vodljivost tla (S/m).

whereas: $\vec{r}'(x_1', x_2'$ and $x_3')$, $\vec{r}(x_1, x_2$ and $x_3)$, are coordinates of the field source and the observation point, dl is the path of integration, $R(\vec{r},\vec{r}')$ is the distance between the source and the observation point, and κ is the specific electrical conductivity of the soil (S/m).

Udaljenost $R(\vec{r},\vec{r}')$ između izvora i točke promatranja u proširenijem zapisu glasi:

The distance of $R(\vec{r},\vec{r}')$ from the source and the observation point in the extended expression reads:

$$R(\vec{r},\vec{r}')=|\vec{r}-\vec{r}'|=\sqrt{(x_1-l')^2+x_2^2+x_3^2}. \tag{2}$$

Uz pretpostavku da je linijska gustoća izvora polja konstantna veličina $\lambda(r') = \mathrm{const} = \lambda$ na putu integracije, izraz (1) može se zapisati kao:

Assuming that the line density of the field source is a constant value $\lambda(r') = \mathrm{const} = \lambda$ on the integration path, expression (1) can be described as:

$$\varphi(r)=\frac{\lambda}{4\pi\kappa}\int_{-c}^{+c}\frac{1}{\sqrt{(x_1-l')^2+x_2^2+x_3^2}}\mathrm{d}l'. \tag{3}$$

Integriranjem izraza (3) i uvrštavanjem donje $-c$ i gornje granice $+c$ dobiva se izraz za potencijal u okolini ravnog tankog vodiča (štapa) koji uključuje i njegovu površinu, a koji glasi:

By integrating expression (3) and adding the lower $-c$ and upper $+c$ border we arrive at the expression for the potential in the surroundings of the straight thin conductor (rod) which includes its surface area and reads:

$$\varphi(x_1,x_2,x_3)=\frac{\lambda}{4\pi\kappa}\ln\frac{x_1+c+\sqrt{(x_1+c)^2+x_2^2+x_3^2}}{x_1-c+\sqrt{(x_1-c)^2+x_2^2+x_3^2}}. \tag{4}$$

Odnosno kako je $i = \lambda l$ izraz (4) može se zapisati kao:

This means that since $i = \lambda l$ expression (4) can be noted as:

$$\varphi(x_1,x_2,x_3)=\frac{i}{4\pi\kappa l}\ln\frac{x_1+c+\sqrt{(x_1+c)^2+x_2^2+x_3^2}}{x_1-c+\sqrt{(x_1-c)^2+x_2^2+x_3^2}}. \tag{5}$$

Otpor rasprostiranja određuje se iz omjera potencijala uzemljivača i struje koja ga je uzrokovala $R_u = \varphi_u / i$. Potencijal uzemljivača najlakše je odrediti u njegovoj sredini, tj. u točki $T(x_1 = 0, x_2 = r, x_3 = 0)$ te se dobiva [3]:

Ground resistance is determined from the ratio of the ground potential and the current that caused it $R_u = \varphi_u / i$. The potential of the ground is easiest to determine at its center i.e. at point $T(x_1 = 0, x_2 = r, x_3 = 0)$, where we obtain [3]:

$$\varphi_u = \frac{i}{4\pi\kappa l} \ln \frac{c + \sqrt{c^2 + r^2}}{-c + \sqrt{c^2 + r^2}} \, . \tag{6}$$

Dijeljenjem izraza (6) sa strujom dobiva se izraz za otpor rasprostiranja usamljenoga cjevastog uzemljivača [3], koji glasi:

By dividing expression (6) by current we arrive at the expression for ground resistance of a lone pipe conductor [3], which reads:

$$R_u = \frac{1}{4\pi\kappa l} \ln \frac{l/2 + \sqrt{(l/2)^2 + r^2}}{-l/2 + \sqrt{(l/2)^2 + r^2}} \, . \tag{7}$$

Kako se potencijal na površini cilindričnog vodiča mijenja od kraja prema sredini za točnije proračune bolje je upotrebljavati srednji potencijal uzemljivača kao potencijal na kojem se nalazi uzemljivač. Srednja vrijednost potencijala na površini uzemljivača glasi:

Since the potential on the surface of the cylindrical conductor is changing from the end towards the middle, it is better to use the mean potential of the ground conductor as the potential on which the ground conductor is located, to obtain more accurate calculations. The mean value of the potentials on the surface of the ground conductor reads:

$$\varphi_{SR} = \frac{i}{4\pi\kappa l^2} \int_{-c}^{+c} \ln \frac{x_1 + c + \sqrt{(x_1 + c)^2 + r^2}}{x_1 - c + \sqrt{(x_1 - c)^2 + r^2}} \, \mathrm{d}x_1 \, . \tag{8}$$

Integracija je u tom slučaju mnogo složenija, a kao rezultat dobiva se:

The integration in this case is much more complex, and the result is:

$$\varphi_{SR} = \frac{1}{4\pi\kappa l^2} \left[l \ln \frac{l + \sqrt{l^2 + r^2}}{-l + \sqrt{l^2 + r^2}} + 2\left(r - \sqrt{r^2 + l^2} \right) \right] . \tag{9}$$

Analognim postupkom, tj. dijeljenjem izraza (9) sa strujom *i* dobiva se izraz za otpor rasprostiranja cjevastog uzemljivača [1]:

In an analogous procedure i.e. by dividing expression (9) by current *i* we arrive at the expression for ground resistance of the pipe conductor [1]:

$$R_u = \frac{1}{4\pi\kappa l} \left[\ln \frac{l + \sqrt{l^2 + r^2}}{-l + \sqrt{l^2 + r^2}} + 2\left(\frac{r}{l} - \sqrt{1 + \left(\frac{r}{l}\right)^2} \right) \right] . \tag{10}$$

Izraz (7) može se smatrati pojednostavnjenjem izraza (10) u slučaju kada je duljina vodiča uzemljivača u odnosu na njegov radijus takva da vrijedi $l / r > 100$. U suprotnom, kod kratkih i debelih cijevi treba upotrebljavati izraz (10), koji inače vrijedi za sve omjere duljine i radijusa vodiča koji se koriste u praksi.

Expression (7) can be seen as a simplified expression (10) when the length of the ground conductor relative to its radius makes $l / r > 100$ applicable. Otherwise, in case of short and thick pipes, expression (10) should be used, which is generally valid for all the ratios between length and radius of the conductor that are used in practice.

2.2 Jednoslojni model tla

U slučaju jednoslojnog modela tla postoji samo jedna granica diskontinuiteta specifične električne vodljivosti, tj. granica tlo-zrak. Utjecaj tog diskontinuiteta na električne prilike u tlu uvažava se primjenom metode odslikavanja. U tu svrhu svaki izvor polja (struje odvoda iz uzemljivača), a time i svi vodiči uzemljivača ako je on složen uzemljivač imaju svoju sliku u području iza granice diskontinuiteta specifične električne vodljivosti, odatle i slijedi naziv te metode. Kako je položaj odslikanih izvora polja lako odrediti, teži je zadatak odrediti njihov iznos. U tu svrhu mogu se primijeniti zaključci iz elektrostatike i odslikavanja na ravnini i teorema prema kojemu se oni prenose na strujna polja uz odgovarajuću zamjenu varijabli. Više detalja o toj problematici i analogiji između strujnih i električnih polja može se pronaći u [2], [4].

Granični uvjeti koje mora zadovoljiti strujno polje prilikom prelaska granice dvaju medija različitih vodljivosti prema [2] i [4] su:

2.2 Single-layer soil model

In case of a single-layer soil model there is only one discontinuity boundary of the specific electrical conductivity i.e. the soil-air boundary. The effect of this discontinuity on the electrical conditions in the soil is taken into account by applying the method of images. To this end each field source (leakage current of the ground counductor), and all the conductors in case of a complex ground conductor type, have their images in the area behind the discontinuity boundary of the specific electrical conductivity, which is how the method got its name. Though the positions of the images of the field sources are easy to determine, it is more difficult to determine their value. To this end the conclusions of electrostatics and the images on the plane and the theorem may be used, according to which they are transferred to current fields with an appropriate exchange of variables. For more details on this subjects and the analogy between the current and the electrical fields see [2], [4].

Boundary conditions to be fulfiled by a current field in crossing the boundary between two media of differing conductivity according to [3] and [4] are:

$$\vec{n}\times\left(\vec{J}_{\mathrm{A}}-\vec{J}_{\mathrm{B}}\right)=0, \tag{11}$$

odnosno:

notably:

$$\vec{n}\cdot\left(\vec{J}_{\mathrm{A}}-\vec{J}_{\mathrm{B}}\right)=0, \tag{12}$$

gdje je:
$\vec{n}$ - vektor normale na ravninu diskontinuiteta električne vodljivosti,
$\vec{J}_{\mathrm{A}}$ - gustoća struje u mediju A na granici diskontinuiteta električne vodljivosti,
$\vec{J}_{\mathrm{B}}$ - gustoća struje u mediju B na granici diskontinuiteta električne vodljivosti.

whereas:
$\vec{n}$ - normal vector of the plane of the discontinuity of the electrical conductivity,
$\vec{J}_{\mathrm{A}}$ - current density in medium A on the discontinuity boundary of the electrical conductivity,
$\vec{J}_{\mathrm{B}}$ - current density in medium B on the discontinuity boundary of the electrical conductivity.

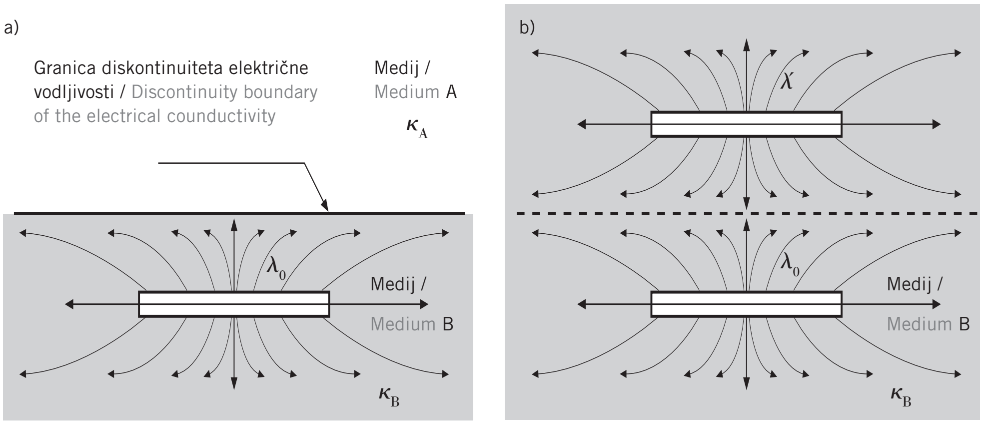
Prema izrazu (11) tangencijalna komponenta strujnog polja na granici diskontinuiteta specifične električne vodljivosti ostaje nepromijenjena, dok prema izrazu (12) normalna komponenata strujnog polja granicu diskontinuiteta prelazi kontinuirano [5].

Za ilustraciju slika 3a prikazuje izgled strujnog polja u okolini cijevi položene paralelno s ravninom koja razdvaja dva medija različitih specifičnih električnih vodljivosti κ_{A} i κ_{B}. Strujno polje u poluprostoru B, tj. mediju B odgovaraju strujnom polju kao da je sav prostor ispunjen medijem B

According to expression (11) the tangential component of the current field on the discontinuity boundary of the specific electrical conductivity remains unchanged, whereas according to expression (12) the normal component of the current field continuously crosses the discontinuity boundary [5].

To illustrate, Figure 3a shows the appearance of the current field around the pipe placed in parallel with the plane dividing two media of different specific electrical conductivity κ_{A} and κ_{B}. The current field in half-space B i.e. medium B corresponds to the te da postoji još jedan izvor polja, koji se nalazi u poluprostoru A, tj. mediju A na udaljenosti od granice diskontinuiteta specifične električne vodljivosti koja odgovara udaljenosti originalnog vodiča od iste (slika 3b). Iz toga proizlazi da se zadaća rješavanja polja u okolini uzemljivača može riješiti tehnikom odslikavanja vodiča na granici diskontinuiteta specifične električne vodljivosti.

current field as if the entire space were filled with medium B and as if there were another field source in half-space A i.e. medium A at the distance from the discontinuity boundary of the specific electrical conductivity which corresponds to the distance of the original conductor from the same (Figure 3b). It follows that the task of dealing with the field around the ground conductor can be fulfilled by the method of images for the conductor on the discontinuity boundary of the specific electrical conductivity.



Slika 3
Izgled strujnog polja u mediju B u slučaju kada on graniči s medijem drugačije vodljivosti (medij A) i ekvivalentni sustav s odslikanim izvorima polja

Figure 3
Appearance of the current field in medium B when it borders on a medium with different conductivity (medium A) and equivalent system with field source images

Rješavanjem (11) i (12) dobiva se izraz za iznos odslikane linijske gustoće struje u području A (medij A), koji glasi:

Solving (11) and (12) we arrive at the expression for the value of images for line density of the current in space A (medium A), which reads:

$$\lambda'_A = \frac{\kappa_B - \kappa_A}{\kappa_B + \kappa_A}\lambda_0 . \tag{13}$$

U tom trenutku valja iskoristiti podatke iz prakse [6], odnosno konkretne vrijednosti specifične električne vodljivosti tla i zraka i uvrstiti ih u prethodni izraz. Specifična električna vodljivost tla kreće se u rasponu od 10^{-2} do 10^{-3} S/m, dok je specifična električna vodljivost zraka reda 10^{-18} S/m. Uvrštavanjem posljednjih podataka u jednadžbu (13) dobiva se:

At this point, we use data from the practice [6] or specific values of the electrical conductivity of soil and air and include them in the expression above. The specific electrical conductivity of the soil ranges from 10^{-2} to 10^{-3} S/m, whereas the specific electrical conductivity of the air is 10^{-18} S/m. By including these data in equation (13) we get:

$$\lambda' = \lambda_0 \frac{10^{-2}/10^{-18} - 1}{10^{-2}/10^{-18} + 1} = \lambda_0 \frac{10^{16} - 1}{10^{16} + 1} \approx \lambda_0 . \tag{14}$$

Za specifičnu električnu vodljivost tla od $\kappa = 0{,}01$ S/m (tipična specifična električna vodljivost tla u Slavoniji) pogreška aproksimacije prema [7] u odnosu na egzaktnu vrijednost iznosi:

For the specific electrical conductivity of $\kappa = 0{,}01$ S/m (typical specific electrical conductivity of the soil in Slavonia) the approximation error according to [7] with regard to the exact value is:

$$p\,\% = \frac{\lambda_0 - \dfrac{10^{16}-1}{10^{16}+1}\lambda_0}{\dfrac{10^{16}-1}{10^{16}+1}\lambda_0}\cdot 100\ \% = 2\cdot 10^{-14}\ \%\,. \tag{15}$$

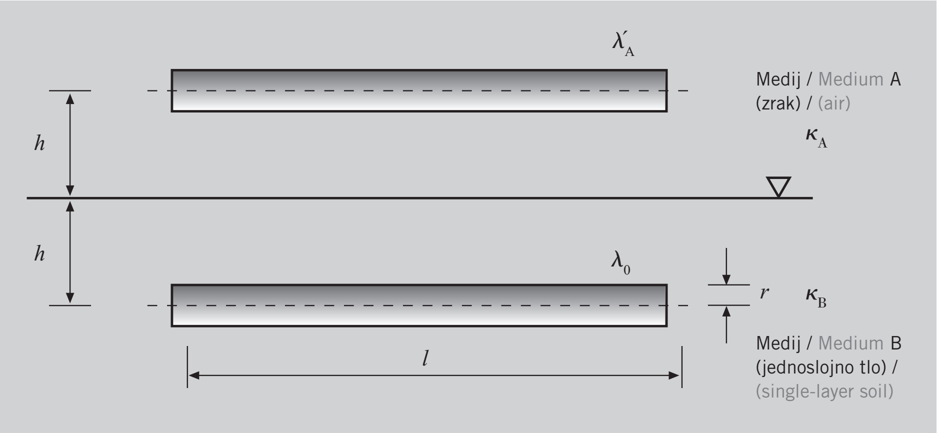
Za specifičnu električnu vodljivost tla od $\kappa = 0{,}001$ S/m (isušeno tlo) pogreška aproksimacije u odnosu na egzaktnu vrijednost iznosi:

For the specific electrical conductivity of $\kappa = 0{,}001$ S/m (dry soil) with regard to the exact value is:

$$p\,\% = \frac{\lambda_0 - \dfrac{10^{15}-1}{10^{15}+1}\lambda_0}{\dfrac{10^{15}-1}{10^{15}+1}\lambda_0}\cdot 100\ \% = 2\cdot 10^{-13}\ \%\,, \tag{16}$$

što je izvrsna aproksimacija jer je pogreška zanemariva i kreće se u rasponu od 10^{-13} % do 10^{-14} %, za sve slučajeve koji mogu nastupiti u praksi, odnosno za raspon specifične električne vodljivosti tla od $\kappa = 0{,}01$ S/m do $\kappa = 0{,}001$ S/m. Dakle, **gotovo bez ikakve pogreške u računu može se koristiti približni izraz** za iznos odslikanog izvora polja iznad tla, prema kojem je $\lambda' = \lambda_0$. Iz prethodno iznesenoga slijedi da je relativno jednostavno formiranje Greenovih funkcija kada je riječ o jednoslojnom tlu. Dobiveni rezultat primjenjiv je za analitičko izvođenje otpora rasprostiranja jednostavnijih uzemljivača, poput cijevi, trake, prstena, ploče itd. Uvažavanje diskontinuiteta specifične električne vodljivosti na granici tlo-zrak na električne prilike u tlu podrazumijeva uvažavanje porasta potencijala uzemljivača nastalog zbog slike vodiča u poluprostoru A (medij A-zrak), što je prikazano slikom 4.

which is an excellent approximation, because the error is negligible, ranging from 10^{-13} % to 10^{-14} % in all the cases that may occur in practice for the range of the specific electrical soil conductivity from $\kappa = 0{,}01$ S/m to $\kappa = 0{,}001$ S/m. Consequently, almost **without any error in the calculation, we may use the approximate expression** for the amount of the field source image above the ground, according to which $\lambda' = \lambda_0$. It follows that setting up Green's functions is relatively simple when it comes to single-layer soil. The result obtained is applicable to the analytical derivation of ground resistance of simpler ground conductors such as pipes, strips, rings, plates etc. Taking into account the discontinuity of the specific electrical conductivity at the boundary between soil and air on the electrical conditions in the soil includes recognising the increase in the potential of the ground due to the image of the conductor in half-space A (medium A-air), as shown in Figure 4.



Slika 4
Dvodimenzionalni prikaz cilindričnog vodoravno ukopanog uzemljivača u jednoslojno anizotropno homogeno tlo

Figure 4
2D rendering of the cylindrical horizontaly buried ground conductor in single-layer anisotropic homogenous soil

Potencijal vodiča uzemljivača φ_u u ovom slučaju jednak je zbroju potencijala koji nastaje na njemu zbog vlastite struje odvoda φ_λ koji je dan izrazima

The potential of the ground conductor φ_u in this case equals the sum of potentials occuring on it because of its own leakage current φ_λ as noted in

(6) i (9) i potencijala nastalog uslijed odslikanog vodiča $\varphi_{\lambda'}$ iznad tla koji se nalazi od aksijalne osi originalnog vodiča na udaljenosti $2h$, tj. vrijedi $\varphi_u = \varphi_\lambda + \varphi_{\lambda'}$. Ukupni potencijal na površini vodiča glasi:

expressions (6) and (9), and the potential occuring on account of the conductor image $\varphi_{\lambda'}$ above the ground at the distance from the axialne axis of the original conductor of $2h$ i.e. $\varphi_u = \varphi_\lambda + \varphi_{\lambda'}$. The total potential on the surface of the conductor reads:

$$\varphi_u = \frac{i}{4\pi\kappa l}\left[\ln\frac{l/2+\sqrt{(l/2)^2+r^2}}{-l/2+\sqrt{(l/2)^2+r^2}}+\ln\frac{l/2+\sqrt{(l/2)^2+(2h)^2}}{-l/2+\sqrt{(l/2)^2+(2h)^2}}\right]. \tag{17}$$

Kada se potencijal na površini vodiča određuje primjenom metode srednjeg potencijala, uvažavajući utjecaj odslikanog izvora polja, taj ukupni potencijal na površini vodiča uzemljivača iznosi:

When the potential on the surface of the conductor is determined by applying the method of mean potential, taking into account the effect of the field source image the total potential on the surface of the ground conductor is:

$$\varphi_u = \frac{i}{4\pi\kappa l}\left[\ln\frac{l+\sqrt{l^2+r^2}}{-l+\sqrt{l^2+r^2}}+\ln\frac{l+\sqrt{l^2+(2h-r)^2}}{-l+\sqrt{l^2+(2h-r)^2}}+2\left(\frac{2h-r}{l}-\sqrt{1+\left(\frac{r}{l}\right)^2}-\sqrt{1+\left(\frac{2h-r}{l}\right)^2}\right)\right]. \tag{18}$$

Dijeljenjem prethodnih dvaju izraza sa strujom i, koja je uzrok potencijalu, dobivaju se izrazi za otpor rasprostiranja vodoravno ukopanog cjevastog uzemljivača za slučaj kada je uvažena granica tlo-zrak. Kako je dvostruka udaljenost od aksijalne osi vodiča do slike ($2h$) puno veća od radijusa vodiča uzemljivača (r), tj. vrijedi $2h >> r$, pojednostavnjeni izrazi kojima se uvažavaju odslikani izvori polja glase:

Dividing the two expressions above by current i, which is the cause of the potential, we arrive at the expressions for ground resistance of a horizontally buried pipe conductor when the soil-air boundary is taken into account. Since the double distance from the axial axis from the conductor to the image charge ($2h$) is much bigger than the radius of the ground conductor (r) i.e. $2h >> r$, the simplified expressions taking into account the field source images read:

$$R_u = \frac{1}{4\pi\kappa l}\left[\ln\frac{l/2+\sqrt{(l/2)^2+r^2}}{-l/2+\sqrt{(l/2)^2+r^2}}+\ln\frac{l/2+\sqrt{(l/2)^2+(2h)^2}}{-l/2+\sqrt{(l/2)^2+(2h)^2}}\right], \tag{19}$$

$$R_u = \frac{1}{4\pi\kappa l}\left[\ln\frac{l+\sqrt{l^2+r^2}}{-l+\sqrt{l^2+r^2}}+\ln\frac{l+\sqrt{l^2+(2h)^2}}{-l+\sqrt{L^2+(2h)^2}}+2\left(\frac{2h}{l}-\sqrt{1+\left(\frac{r}{l}\right)^2}-\sqrt{1+\left(\frac{2h}{l}\right)^2}\right)\right], \tag{20}$$

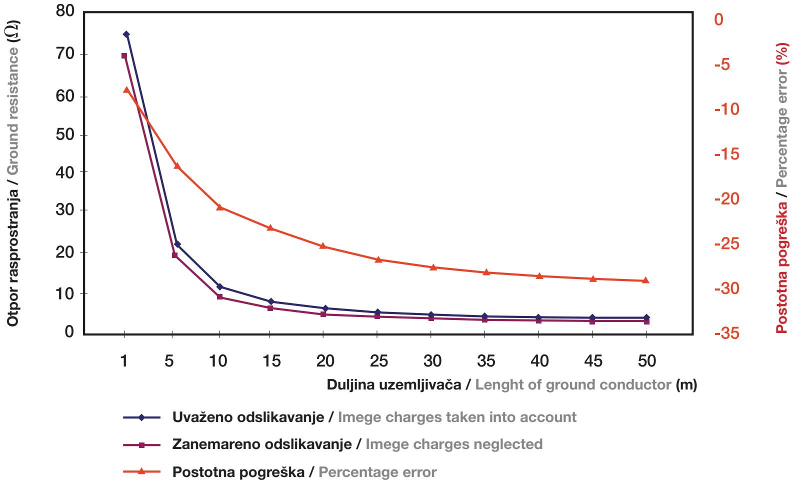
gdje je:
l - aksijalna duljina uzemljivača (m),
r - radijus (polumjer) uzemljivača (m),
κ - specifična električna vodljivost tla (S/m),
h - dubina ukopavanja uzemljivača (m).

whereas:
l - axial length of ground conductor (m),
r - radius of ground conductor (m),
κ - specific electrical conductivity of the soil (S/m),
h - depth to which the ground conductor is buried (m).

Na slici 5 grafički je prikazan otpor rasprostiranja horizontalno ukopanoga cjevastog uzemljivača u slučajevima kada je uvažena granica diskontinuiteta specifične električne vodljivosti tlo-zrak (plava

Figure 5 graphically shows the ground resistance of a horizontally buried pipe conductor when the discontinuity boundary of the specific electrical conductivity between soil and air (blue curve) is krivulja) i u slučaju kada je ista zanemarena (ružičasta krivulja). Otpor rasprostiranja određen je izrazima (10) i (20). Ujedno je prikazana i postotna pogreška koja nastaje zanemarivanjem granice tlo-zrak (crvena krivulja).

taken into account, and when it is not (pink curve). Ground resistance is determined by expressions (10) and (20). Figure 5 also shows the percentage error which occurs when the soil-air boundary is neglected (red curve).



Slika 5
Otpor rasprostiranja horizontalno ukopanoga cjevastog uzemljivača u funkciji njegove duljine s utjecajem i bez utjecaja granice tlo-zrak i postotna pogreška u njezinu zanemarivanju

Figure 5
Ground resistance of a horizontally buried pipe conductor considering its length with and without the effect of the soil-air boundary, and percentage error in case of neglecting the same

Tablica 1 - Otpor rasprostiranja horizontalno ukopanoga cjevastog uzemljivača s utjecajem i bez utjecaja granice tlo-zrak i postotna pogreška u njezinu zanemarivanju
Table 1 - Ground resistance of a horizontally buried pipe conductor with and without the effect of the soil-air boundary, and percentage error in case of neglecting the same

Duljina vodiča / Conductor length (m)	Otpor rasprostiranja*/Ground resistance (Ω)		Pogreška / Error (%)
	Zanemarena granica tlo-zrak / Neglected soil-air boundary	Uvažena granica tlo-zrak / Soil-air boundary taken into account	
1	68,569	74,041	-7,392
10	10,507	13,363	-21,371
50	2,613	3,663	-28,648
100	1,417	2,050	-30,869

* Radijus vodiča je 0,01 m, a dubina ukopavanja 0,7 m. / The radius of the conductor is 0,01 m, and the depth to which it is buried is 0,7 m.

Iz slike 3 i tablice 1 lako je zapaziti da porastom duljine uzemljivača u odnosu na dubinu ukopavanja raste i pogreška ako se zanemari diskontinuitet specifične električne vodljivosti na granici tlo-zrak. Kod dugih uzemljivača pogreška je veća od 30 %. Ujedno je važno napomenuti da je pogreška negativna, što je otežavajuća okolnost jer ako se zanemari odslikavanje, uzemljivač će biti poddimenzioniran, odnosno imat će prevelik otpor rasprostiranja, što rezultira skupim postupcima sanacije u praksi. Iz toga je lako zaključiti da se granica tlo-zrak mora uvažavati u svim slučajevima koji mogu nastupiti u praksi.

Figure 3 and Table 1 clearly show that with the increase in the length of the ground conductor in relation to the depth to which the conductor is buried the error also increases if the discontinuity of the specific electrical conductivity at the soil-air boundary is neglected. In long ground conductors the error exceeds 30 %. It is also important to note that the error is negative, which is an aggravating circumstance, because if image charge is neglected the ground conductor will be undersized, i.e. it will have too big ground resistance, which results in expensive rehabilitation procedures in practice. Consequently, the soil-air boundary must be considered in the cases that may appear in practice.

Svaka važnija studija ili proračun uzemljivača zahtijevat će uvažavanje i višestruke vertikalne promjene specifične električne vodljivosti tla. Vertikalna promjena specifične električne vodljivosti tla, može biti uzrokovana: isušivanjem površinskog sloja tla ljeti, zaleđivanjem površinskog sloja zimi, različitim kemijskim sastavom, različitom granulacijom, posipavanjem tla šljunkom ili asfaltom (u postrojenjima).

Bez obzira na narav uzroka promjene specifične električne vodljivosti, takvi se slučajevi najčešće rješavaju primjenom metode višestrukih odslikavanja. U tu svrhu koriste se prenamijenjeni prethodno izvedeni izrazi za iznos i položaj odslikanih linijskih gustoća struje u dvoslojnom vodljivom mediju. Tehnički zanimljivi slučajevi nastupaju kada se (u prethodnom slučaju) uzemljivač nalazi iznad sloja koji ima mali specifični električni otpor tla, tj. veliku specifičnu električnu vodljivost, tj. kada je površinski sloj tla isušen ljeti ili zaleđen zimi, te u drugom slučaju kada se on nalazi u sloju s manjim specifičnim električnim otporom tla, tj. većom specifičnom vodljivosti, tj. kada je uzemljivač u krškom predjelu gdje je velik specifični električni otpor tla, a u rov s uzemljivačem nanesena je zemlja povoljnije vodljivosti te u slučajevima kada je specifična električna otpornost tla gornjeg sloja niža od specifičnog električnog otpora tla donjeg sloja. Oba navedena slučaja analiziraju se primjenom dvoslojnog modela tla, prikazanom u sljedećem poglavlju.

2.3 Dvoslojni model tla - uzemljivač u gornjem sloju tla

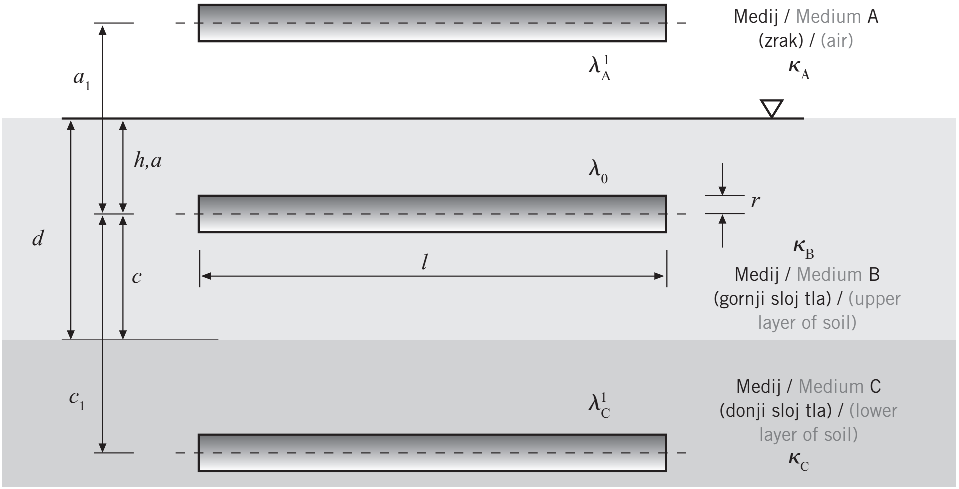
U ovom poglavlju detaljno je prikazan zanimljiv i tehnički veoma važan slučaj. Neka je izvor polja (linijska gustoća struje λ_0) postavljen u sloju tla B, koji razdvaja dva poluprostora-tla različitih specifičnih električnih vodljivosti, kao što je to prikazano slikom 6. Potrebno je odrediti iznose i položaje odslikanih linijskih gustoća struje tako da se može s visokom točnosti odrediti iznos skalarnog električnog potencijala u području u kojemu se nalazi izvor (područje B). Tri medija različitih specifičnih električnih vodljivosti označeni su slovima A, B i C, što odgovara specifičnim električnim vodljivostima medija: κ_A, κ_B i κ_C. Izvor polja je linijska gustoća struje λ_0, koja se nalazi na udaljenosti a od poluprostora A (zrak), odnosno na udaljenosti c od poluprostora C. Iznosi i položaji odslikanih linijskih gustoća struja u poluprostorima A i C određuju se uz pomoć izraza za slučaj kada ravnina razdvaja dva poluprostora različitih specifičnih električnih vodljivosti [8].

Every major study or computation of the ground conductor will also require taking into account the multiple vertical change in the speciifc electrical conductivity of the soil. The vertical change in the specific electrical conductivity of the soil may be caused by: drying of the surface layer in the summer, freezing of the surface layer in the winter, different chemical composition, different granulation, covering the soil with gravel or asphalt (at plant facilities).

Notwithstanding the nature of the cause of the change in the specific electrical conductivity, such cases are most frequently dealth with by using the method od multiple images. To this end the previously derived expressions for the amount and position of image line density of the current in a double-layer medium is used. Technically interesting instances occur when in the aforementioned case the ground conductor is placed above the soil with low specific soil resistivity i.e. high specific electrical conductivity, that is when the surface layer of the soil is dried up in the summer or frozen in the winter, and in another case, when the conductor is placed in a layer with low specific soil resistivity i.e. with a higher specific conductivity, notably in a karstic area where there is a great specific soil resistivity, and the trench with the ground conductor is filled with earth of more favourable conductivity. Also in the cases when the specific soil resistivity of the upper layer is lower than the specific soil resitivity of the lower layer. Both these cases can be analysed by using a double-layer soil model, presented in the following section.

2.3 Double-layer soil model - ground conductor in the upper soil layer

This section describes an interesting and technically very important case which will be dealt with in detail. Let us assume that the field source (line density of the current λ_0) is placed in the B layer of the soil which divides two half-spaces - the soils with different specific eletrical conductivity, as shown in Figure 6. It is necessary to determine the amounts and positions of image line densities so that it is possible to determine with high accuracy the amount of scalar electrical potential in the area in which the source is positioned (area B). Three media with differing specific electrical conductivity are marked by the letters A, B and C, corresponding to the specific electrical conductivity of the media: κ_A, κ_B and κ_C. Field source is the current density λ_0 which is placed at the distance a from the half-space A (air), i.e. at the distance c from the half-space C. The amounts and positions of the image line densities of the currents in half-spaces A and C are determined by means of the expression for the case when the plane divides two half-spaces of different specific electrical conductivities [8].



Slika 6
Dvodimenzionalni prikaz ukopanog cjevastog uzemljivača
Figure 6
2D rendering of buried pipe conductor

Iz izraza za iznose odslikanih linijskih gustoća struja

From the expressions for the amounts of line density images of currents

$$\lambda_A' = \frac{\kappa_B - \kappa_A}{\kappa_B + \kappa_A}\lambda_0 . \tag{21}$$

Slike linijske gustoće struje koje nastaju u području A (A-zrak) imat će indeks A dolje, tj. λ_A, a redni broj slike (k) u tom području označavat će se indeksom gore $\lambda^{(k)}$ pa će potpuno indeksiranje odslikanih linijskih gustoća struja koje se nalaze u području A imati oznaku $\lambda_A^{(k)}$. Isto vrijedi i za područje C, te je k-ta slika u području C prikazana kao $\lambda_C^{(k)}$.

Prve slike originalnog izvora polja u područjima A i C imaju sljedeće iznose i položaje u odnosu na vodič uzemljivača:

The images of line density of the current occuring in area A (A-air) will have lower index A i.e. λ_A, and the ordinal number of the image (k) will in such a case be marked by the upper index $\lambda^{(k)}$, so that full indexing of line density images of currents in area A will be marked $\lambda_A^{(k)}$. The same applies to area C, and the k image in area C is represented as $\lambda_C^{(k)}$.

The first images of the original field source in the areas A and C have the following amounts and positions in relation to the ground counductor:

$$\lambda_A^{(1)} = \frac{\kappa_B - \kappa_A}{\kappa_B + \kappa_A}\lambda_0 = \alpha\lambda_0 , \tag{22}$$

a udaljenost od aksijalne osi uzemljivača iznosi $a_1 = 2a$,

and the distance from the axial axis of the ground conductor is $a_1 = 2a$,

$$\lambda_C^{(1)} = \frac{\kappa_B - \kappa_C}{\kappa_B + \kappa_C}\lambda_0 = \beta\lambda_0 , \tag{23}$$

a udaljenost od aksijalne osi uzemljivača iznosi $c_1 = 2c$.

and the distance from the axial axis of the ground conductor is $c_1 = 2c$.

Ponavljanjem postupka za $\lambda_A^{(1)}$ i $\lambda_C^{(1)}$ dobivaju se položaji i iznosi slika linijskih gustoća struje drugoga reda:

$$\lambda_A^{(2)} = \alpha\, \lambda_C^{(1)} = (\alpha\beta)\, \lambda_0,$$

$$a_2 = 2c_1 = 4c,$$

$$\lambda_C^{(2)} = \beta\lambda_A^{(1)} = (\beta\alpha)\, \lambda_0,$$

$$c_2 = 2a_1 = 4a.$$

Analognim postupkom za treću iteraciju $k = 3$ dobiva se:

$$\lambda_A^{(3)} = \alpha\, \lambda_C^{(2)} = \alpha(\beta\alpha)\, \lambda_0,$$

$$a_3 = 2b_2 = 2 \cdot (4a) = 8a,$$

$$\lambda_C^{(3)} = \beta\lambda_A^{(2)} = \beta(\alpha\beta)\lambda_0,$$

$$c_3 = 2a_2 = 2 \cdot (4c) = 8c.$$

Na osnovi tih triju iteracija može se odrediti sljedeća shema prikazana slikom 7 prema kojoj se mogu određivati iznosi odslikanih linijskih gustoća struje i njihove udaljenosti od koordinatnog početka (položaj izvora polje-uzemljivač).

Repeating the procedure for $\lambda_A^{(1)}$ and $\lambda_C^{(1)}$ we obtain the positions and amounts of the images of line densities of the currents of the second order:

$$\lambda_A^{(2)} = \alpha\, \lambda_C^{(1)} = (\alpha\beta)\, \lambda_0,$$

$$a_2 = 2c_1 = 4c,$$

$$\lambda_C^{(2)} = \beta\lambda_A^{(1)} = (\beta\alpha)\, \lambda_0,$$

$$c_2 = 2a_1 = 4a.$$

By an analogous procedure for a third iteration $k = 3$ we obtain:

$$\lambda_A^{(3)} = \alpha\, \lambda_C^{(2)} = \alpha(\beta\alpha)\, \lambda_0,$$

$$a_3 = 2b_2 = 2 \cdot (4a) = 8a,$$

$$\lambda_C^{(3)} = \beta\lambda_A^{(2)} = \beta(\alpha\beta)\lambda_0,$$

$$c_3 = 2a_2 = 2 \cdot (4c) = 8c.$$

On the basis of these three iterations we can determine the next schematic shown in Figure 7 according to which we can determine the amounts of the images of current line density and their distance from the coordinant start (position of the field source-ground conductor).

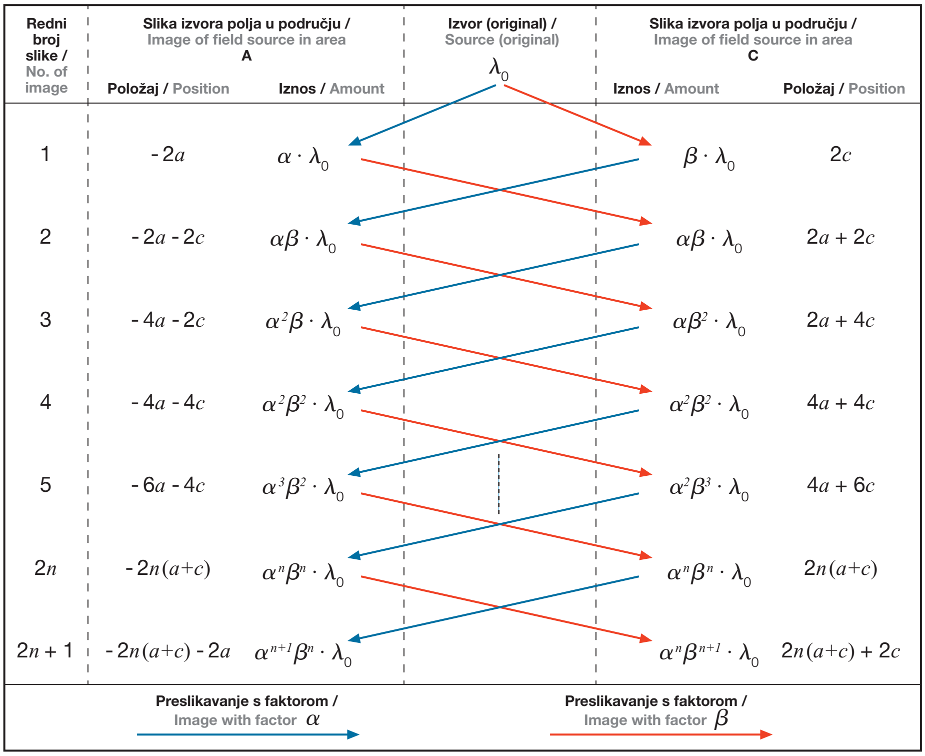


Figure 7
Shematski prikaz određivanja faktora odslikavanja i udaljenosti slike vodiča od aksijalne osi originalnog vodiča uzemljivača
Figure 7
Schematic of the image factor and distance of the conductor image from the axial axis of the original ground conductor

Primjenom zaključka iz točke 2.2 slijedi da je koeficijent $\alpha = 1$. Otpor rasprostiranja horizontalnog cjevastog uzemljivača ukopanog u dvoslojno tlo uvažavajući slike do drugog reda glasi:

On the basis of the conclusion in section 2.2 it is easy to find that the coefficient $\alpha = 1$. Ground resistance for a horizontal pipe conductor buried in a two-layer soil, considering the images up to the second order, reads:

$$R_u \approx \frac{1}{4\pi\kappa_B l}\left[(f_1+g_1)+\alpha(f_2+g_2)+\beta(f_3+g_3)+2\alpha\beta(f_4+g_4)\right], \tag{24}$$

gdje su funkcije f i g dane izrazima:

with the functions f and g given in expressions:

$$f_1 = \ln\frac{l+\sqrt{l^2+r^2}}{-l+\sqrt{l^2+r^2}}, \tag{25}$$

$$g_1 = 2\left(\frac{r}{l}-\sqrt{1+\left(\frac{r}{l}\right)^2}\right), \tag{26}$$

$$f_2 = \ln\frac{l+\sqrt{l^2+4a^2}}{-l+\sqrt{l^2+4a^2}}, \tag{27}$$

$$g_2 = 2\left(\frac{2a}{l}-\sqrt{1+\left(\frac{2a}{l}\right)^2}\right), \tag{28}$$

$$f_3 = \ln\frac{l+\sqrt{l^2+4c^2}}{-l+\sqrt{l^2+4c^2}}, \tag{29}$$

$$g_3 = 2\left(\frac{2c}{l}-\sqrt{1+\left(\frac{2c}{l}\right)^2}\right), \tag{30}$$

$$f_4 = \ln\frac{l+\sqrt{l^2+4(a+c)^2}}{-l+\sqrt{l^2+4(a+c)^2}}, \tag{31}$$

$$g_4 = 2\left(\frac{2(a+c)}{l}-\sqrt{1+\left(\frac{2(a+c)}{l}\right)^2}\right). \tag{32}$$

Primjena prethodnih izraza može se ilustrirani numeričkim primjerom.

Here is a numerical example as an illustration of the expressions referred to above.

Neka se vodič trakastog uzemljivača ekvivalentnog radijusa 1,5 cm i duljine 50 m nalazi u gornjem sloju tla kojemu se specifični električni otpor mijenja sezonski te se kreće od 200-500 Ωm, a debljina sloja na mjestu polaganja uzemljivača približno je 1 m. Uzemljivač je ukopan na 0,7 m ispod površine tla. Donji sloj tla nije podložan u većoj mjeri sezonskim varijacijama otpora na mjestu ukopavanja te mu je specifična otpornost približno 100 Ωm. Potrebno je odrediti otpor rasprostiranja uzemljivača pri nepovoljnim sezonskim varijacijama otpora površinskog tla.

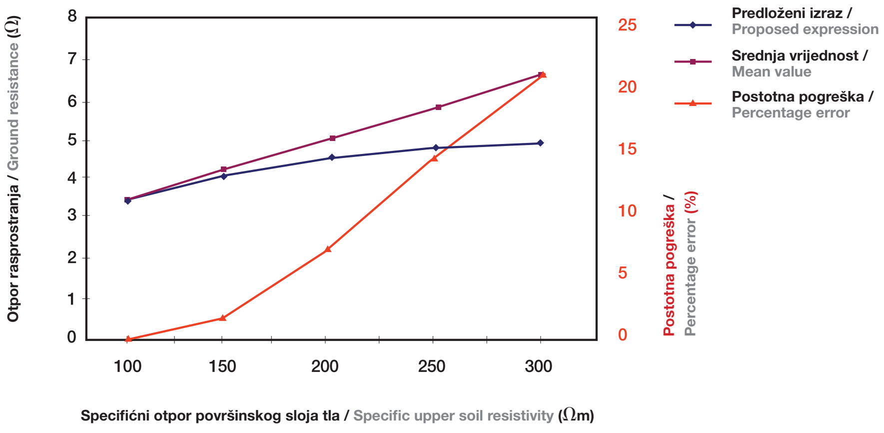
Bez tehnike višestrukog odslikavanja procjenilo bi se otpor rasprostiranja uzemljivača u slučaju da je sve tlo specifičnog električnog otpora 300 Ωm. Tada se uvažavanjem odslikavanja na granici tlo zrak prema izrazu (20) dobiva za otpor rasprostiranja cijevi R_u = 10,601 Ω. Odnosno u slučaju da je sve tlo specifičnog električnog otpora 100 Ωm, otpor rasprostiranja iznosi R_u = 3,534 Ω. Stvarni otpor rasprostiranja bio bi manji od 10,601 Ω, a veći od 3,534 Ω. Mogla bi se uzeti srednja vrijednost koja bi iznosila R_u = 7,068 Ω. Točna vrijednost otpora prema (24) iznosi R_u = 5,812 Ω.

Pogreška procijenjene vrijednosti iznosi 21,6 %, gdje je za točnu vrijednost uzet otpor rasprostiranja od R_u = 5,812 Ω. To je nedopustivo velika pogreška, te je ujedno na tom numeričkom primjeru opravdana uporaba izraza (24), koji uvažava dvoslojnu strukturu tla. Na slici 8 grafički je prikazana ovisnost otpora rasprostiranja u dvoslojnom tlu dobivenog prema izrazu (24), koji se predlaže za uporabu u takvom slučaju i procijenjene (srednje) vrijednosti otpora rasprostiranja određene na način prikazan u numeričkom primjeru.

A strip conductor with the equivalent radius of 1,5 cm and 50 m length is placed in the upper layer of the soil whose specific electrical resistivity changes with the seasons, ranging from 200 to 500 Ωm, and the thickness of the layer at the point where the ground conductor is buried is approximately 1 m. The ground conductor is buried to 0,7 m under the surface. The lower layer of the soil is not subject to major seasonal variations in resistivity at the point where the conductor is buried, and its specific resistivity is approximately 100 Ωm. It is necessary to determine ground resistance for unfavourable seasonal variations of the upper soil resistivity.

Without the technique of multiple images we would estimate ground resistance assuming the entire soil to have the specific electrical resistivity of 300 Ωm. Then taking into account the images on the soil-air boundary according to expression (20), the pipe ground resistance of R_u = 10,601 Ω is obtained. In case the entire soil has the specific electrical resistivity of 100 Ωm, the ground resistance is R_u= 3,534 Ω. The actual ground resistance would be less than 10,601 Ω and greater than 3,534 Ω. We could take the mean value which would be R_u = 7,068 Ω. The exact value of resistance according to (24) is R_u = 5,812 Ω.

The error of the estimated value is 21,6 %, with ground resistance R_u = 5,812 Ω taken as the accurate value, which is an impermissibly big error, so in this numerical example it is justified to use expression (24) which takes into account the two-layer soil structure. Figure 8 shows the dependence of the ground resistance in a two-layer soil obtained from expression (24) which we propose to use in such a case, and the estimated (mean) value of ground resistance determined in the manner shown in the numerical example.



Slika 8
Otpor rasprostiranja vodoravno ukopane trake u funkciji specifične otpornosti površinskog sloja tla za jednoslojni i dvoslojni model tla i postotna pogreška

Figure 8
Ground resistance of a horizontally buried strip conductor considering the specific upper soil resistivity in a single-layer and in a double-layer soil model, and percentage error.

Valja naglasiti da pogreška koja nastaje zanemarivanjem dvoslojne strukture tla ovisi u prvom redu o **omjeru otpora gornjeg i donjeg sloja** tla i veća je što je navedeni omjer veći, a manje ovisi o dubini ukopavanja i debljini površinskog sloja. Slike višeg reda (>2) nije potrebno uvažavati ako se ne traži veća točnost, ali je to u praksi nepotrebno, jer će korekcija biti neznatna u odnosu na nesigurnost mjernih podataka o otporu tla i debljini površinskog sloja tla. Na osnovi slike 8 i numeričkog primjera lako je zaključiti da ako se očekuje znatnija vertikalna promjena specifičnog električnog otpora tla, tada se **obvezno mora uvažavati** dvoslojna struktura tla.

3 ZAKLJUČAK

Temeljem dobivenih rezultata otpora rasprostiranja jednostavnog vodoravno ukopanog cjevastog uzemljivača uporabom različitih modela tla može se zaključiti da minimalno korektni proračun otpora rasprostiranja uzemljivača podrazumijeva jednoslojni model tla, koji govoreći tehnikom odslikavanja uključuje originalni vodič i njegovu sliku iznad tla. Pogreška određivanja otpora rasprostiranja uzemljivača koja bi nastupila zanemarivanjem tog jednostavnog odslikavanja u praktičnim slučajevima kod dugih trakastih uzemljivača dosegnula bi nedopustivo visoku razinu od preko 30 %.

Nadalje, kako je u praksi nerealno očekivati jednoslojno tlo, za točnije proračune i u slučajevima kada se očekuju velike razlike između specifičnih otpornosti gornjeg i donjeg sloja tla obavezna je uporaba dvoslojnog modela tla. Pritom je u većini praktičnih slučajeva dovoljno uvažiti slike vodiča do drugog reda s obiju strana granica diskontinuiteta električne vodljivosti. Pogreška koja bi nastupila prilikom proračuna otpora rasprostiranja u slučaju tla s velikom razlikom specifične otpornosti gornjeg i donjeg sloja kreće se do oko 20 % u većini praktičnih slučajeva. Iako je u ovom radu prikazan slučaj kada je gornji sloj tla s manjom specifičnom otpornosti isti, zaključci vrijede i u obrnutom slučaju. U radu je na praktički najjednostavnijem primjeru uzemljivača prikazana implementacija metode odslikavanja, analizirana točnost modela, a analogni isti postupci mogu se koristiti i za složenije uzemljivače poput zvjezdastog, prstenastog, mrežastog itd., što će biti prikazano u idućem radu.

Prilikom analize utjecaja višeslojnosti tla na složenije tipove uzemljivača, a primjenom rezultata prikazanih u ovom radu, potrebno je uspoređivati najistaknutiju dimenziju složenog uzemljivača

It needs to be said here that the error occurring in neglecting the two-layer soil structure primarily depends on the **ratio between the upper layer resistivity and the lower layer resistivity**, increasing with the ratio, and less on the depth to which the conductor is buried and the thickness of the surface layer. The images of a higher order (>2) need not be taken into account if no particular accuracy is required, and in practice this is unneccessary because the correction will be insignificant compared with the uncertainty of the measurement data of ground resistance and thickness of the upper layer of the soil. On the basis of Figure 8 and the numercial example it is easy to conclude that if a substantial vertical change in the specific soil resistivity is expected, then **it is obligatory to take into account** the double-layer soil structure.

3 CONCLUSION

On the basis of the ground resistance results obtained for a simple horizontally buried pipe conductor by using various soil models, we may conclude that a minimum correct computation of ground resistance applies to the single-layer soil model, which in terms of the technique of images includes the original conductor and its image above the ground. The error in determining ground resistance which would occur if this simple images were neglected in practical cases with long strip conductors would reach the impermissibly high level of more than 30 %.

Furthermore, since it is unrealistic to expect single-layer soil in practice, for more accurate computations and in cases when big differences are expected between the specific resistivity of the upper and the lower layers of the soil it is obligatory to use the double-layer model of the soil. In most cases in practice it suffices to take into account the conductor images up to the second order on both sides of discontinutity boundaries of the electrical conductivity. The error that would occur in the computation of the ground resistance in the soil with a high difference in the specific resistivity between the upper and the lower soil layers reaches up to about 20 % in most practical cases. Although this article has shown a case when the upper layer of the soil has lower specific soil resistivity, the same conclusion applies in the opposite case. The article shows the implementation of the method of images practically on the simplest example of the ground conductor, analysing the accuracy of the model, and analogous procedures can be used for more complex ground conductors such as star, ring and mesh type conductors etc., which will be presented in the article to follow.

koja je u ravnini s ravninom diskontinuiteta otpora s debljinom slojeva tla, te dubinom ukopavanja [6]. U ovom radu namjerno su prikazani izvodi izraza kako bi se čitateljima koji razvijaju vlastita softverska rješenja primjenom metode momenata (MoM) omogućila lakša implementacija krajnjih rezultata ovoga članka, tj. iznosa i položaja odslikanih izvora polja (vodiča) u njihov softver. Više detalja o numeričkim postupcima pri proračunu otpora rasprostiranja zainteresirani čitatelj može naći u [9], [10], [11] i [12]. Prednosti i nedostaci pojedinih numeričkih postupaka opisani su u [8]. Autori ovog rada pripremaju članak u kojem će biti prikazana implementacija dobivenih izraza za proračune složenijih uzemljivača u višeslojnom tlu primjenom metode momenata. Želja je autora potaknuti inženjere u praksi na uporabu složenijih tehnika odslikavanja.

In analysing the effect of multiple layers on more complex types of ground conductors, and using the results shown in this article, it is necessary to compare the biggest dimension of a complex ground conductor which is level with the plane of resistance discontinuity following the thickness of soil layers and the depth to which the conductor is buried [6]. In this article we have deliberately shown the derivations of the expressions to enable the readers who are developing their own software solutions by applying the method of moments (MoM) to more easily implement the end results from this article i.e. the amounts and positions of field source (conductor) images in their software. More details about numerical procedures in the computation of ground resistance can be found in [9], [10], [11] and [12]. The pros and cons of specific numerical procedures are described in [8]. The authors of this article are preparing an article in which they will show the implementation of the expressions obtained for the computations involving more complex ground conductors in a multi-layer soil by applying the method of moments. The wish of the authors is to encourage engineers to use more complex techniques of images in practice.

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