3D PRORAČUN KVAZISTATIČKOG MAGNETSKOG POLJA OKO VODIČA I FEROMAGNETSKE PLOČE INTEGRALNIM JEDNADŽBAMA

THE 3D CALCULATION OF THE QUASISTATIC MAGNETIC FIELD AROUND A CURRENT CARRYING CONDUCTOR AND FERROMAGNETIC PLATE BY MEANS OF INTEGRAL EQUATIONS

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Za model vodiča i feromagnetske ploče napravljen je 3D proračun magnetskog polja pomoću integralnih jednadžbi za kvazistatički slučaj. Uspoređeni rezultati proračuna i mjerenja dobro se slažu. Pristup preko integralnih jednadžbi pokazuje se vrlo efikasnim u prostoru u kojem se traži polje daleko od izvora i u kojem nema puno različitih feromagnetskih materijala. Pri tome se pretpostavlja da su vodljivost i permeabilnost feromagnetskog materijala konstantni. Također je prikazan utjecaj feromagnetskog materijala na magnetsko polja vodiča.

The 3D calculation of the magnetic field around a model of a current carrying conductor and ferromagnetic plate has been performed by means of integral equations in the quasistatic case. There is good agreement between the calculated and measured results. The integral equation approach is very effective in a space in which a field distant from the source is calculated and there are not many different types of ferromagnetic materials. The conductivity and permeability of the ferromagnetic materials are assumed to be constant. Furthermore, the influence of the ferromagnetic materials on the magnetic field of a conductor is shown.

Ključne riječi: 3D proračun, feromagnetski materijal, integralne jednadžbe, kvazistatičko magnetsko polje

Key words: 3D calculation, ferromagnetic material, integral equations, quasistatic magnetic field

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1 UVOD

Pri numeričkom proračunu magnetskog polja metodom konačnih elemenata u prostoru u kojem se traži polje daleko od izvora, može se pojaviti problem vrlo velikog broja potrebnih elemenata za modeliranje kao i problem definiranja rubnih uvjeta na granici domene proračuna. Nasuprot tome u pristupu preko integralnih jednadžbi potrebno je modelirati samo feromagnetske materijale i ne treba postavljati rubne uvjete, jer je domena proračuna beskonačna. čitav se prostor promatra kao slobodni ($\mu=\mu_0$) u kojem postoje primarni izvori (stvarni izvori) i sekundarni izvori (izvori koji modeliraju materijal).

Prikazat će se 3D proračun magnetskog polja preko integralnih jednadžbi na najjednostavnijem modelu; tanki vodič kroz koji teče struja, a u čijoj se blizini nalazi feromagnetska ploča u obliku kvadra.

1 INTRODUCTION

In the numerical calculation of a magnetic field by means of the commonly used finite element method in a space in which a field distant from the source is being sought, problems in connection with the very large number of elements required for modeling and the definition of boundary conditions may arise. However, when using integral equations, only the ferromagnetic materials need to be modeled and it is not necessary to set boundary conditions, since the domain is infinite. All the space in which the primary (real sources) and secondary sources exist is regarded as free ($\mu=\mu_0$).

The 3D calculation of a magnetic field by means of integral equations will be shown for the simplest model; a current carrying conductor and nearby ferromagnetic cuboid plate.

2 MODEL ZA 3D PRORAČUN

Na slici 1 prikazan je ispitivani model, a na slici 2 pojednostavljeni model za proračun u kojem su zanemareni progib i promjer vodiča.

Struja I_0 frekvencije f teče kroz vodič duljine l u smjeru $+x$ osi. Ispod vodiča simetrično je postavljena feromagnetska ploča stalne specifične vodljivosti γ i stalne permeabilnosti μ. Ishodište koordinatnog sustava se postavlja u središte vodiča prema slikama 1 i 2.

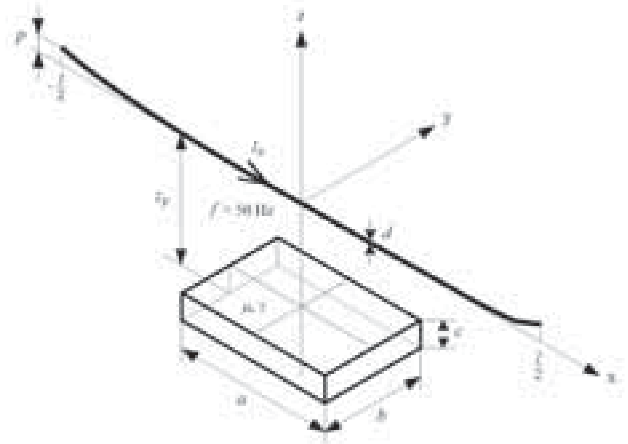
Zanemaruje se utjecaj svih ostalih izvora i materijala na polje promatranog vodiča osim same feromagnetske ploče. Polje će se računati samo u smjeru osi z .

2 MODEL FOR 3D CALCULATION

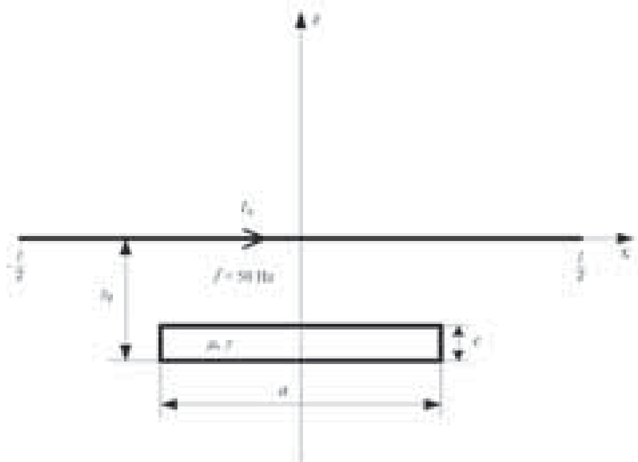
The model studied is shown in Figure 1, while a simplified model for calculation is shown in Figure 2, in which the deflection and diameter of the conductor are ignored.

Current I_0 of frequency f flows through the conductor of length l in the direction of the $+x$ axis. A ferromagnetic plate is placed symmetrically beneath the conductor. The plate has constant conductivity γ and constant permeability μ. The origin of the coordinate system is in the center of the conductor, according to Figures 1 and 2.

The influence of other sources and materials (except the ferromagnetic plate) on the magnetic field of the conductor is ignored. The field will only be calculated along the z axis.



Slika 1
Ispitivani model vodiča i feromagnetske ploče
Figure 1
Studied model of a conductor and ferromagnetic plate



Slika 2
Pojednostavljeni model za proračun magnetskog polja
Figure 2
Simplified model for magnetic field calculation

I_0 – efektivna vrijednost struje koja teče kroz vodič,
f – frekvencija,
l – duljina vodiča,
p – progib vodiča,
d – promjer vodiča,
a – duljina feromagnetske ploče u smjeru x-osi,
b – duljina feromagnetske ploče u smjeru y-osi,
c – duljina feromagnetske ploče u smjeru z-osi,
z_p – z koordinata donje plohe feromagnetske ploče,
γ – vodljivost feromagnetske ploče,
μ – permeabilnost feromagnetske ploče ($\mu=\mu_0 \cdot \mu_r$),
μ_0 – permeabilnost zraka ($\mu_0 = 1{,}256 \cdot 10^{-6}$ H/m).

Zbog kvazistatičnosti polja (f= 50 Hz), proračun će se provesti u kompleksnom području pa će sve vremenski ovisne veličine imati točku iznad naziva (fazor). Ako veličina ima i vektorski karakter, onda će **naziv** biti **podebljan** (**bold**) (npr. $\dot{\boldsymbol{H}}$).

I_0 – effective value of current flowing through conductor,
f – frequency,
l – length of conductor,
p – deflection of conductor,
d – diameter of conductor,
a – length of the ferromagnetic plate in the x-axis direction,
b – length of the ferromagnetic plate in the y-axis direction,
c – length of the ferromagnetic plate in the z-axis direction,
z_p – z-coordinate of the bottom surface of the ferromagnetic plate,
γ – conductivity of the ferromagnetic plate,
μ – permeability of the ferromagnetic plate ($\mu=\mu_0 \cdot \mu_r$)
μ_0 – permeability of the air ($\mu_0 = 1{,}256 \cdot 10^{-6}$ H/m)

Because the field is quasistatic (f = 50 Hz), the calculation will be performed in a complex plane so that all time-dependent variables will have a point above the symbol (phasor). If the variable is a vector, it will have a **bold** symbol (e.g. $\dot{\boldsymbol{H}}$).

3 PRORAČUN MAGNETSKOG POLJA VODIČA (BEZ FEROMAGNETSKE PLOČE)

Dogovorno će sve veličine koje uzrokuje primarni izvor (vodič) I_0 imati indeks 0. Vektorski magnetski potencijal A_0 i jakost magnetskog polja H_0 proračunat će se prema Biot-Savartovu zakonu. Budući da struja u vodiču teče samo u smjeru $+x$ osi, u promatranoj točki prostora vektorski magnetski potencijal imat će samo x komponentu, a jakost magnetskog polja y i z komponentu.

Vektorski magnetski potencijal i jakost magnetskog polja u bilo kojoj točki (x,y,z) dani su sljedećim izrazima:

3 CALCULATION OF A_0 AND H_0 FOR A CONDUCTOR (WITHOUT A FERROMAGNETIC PLATE)

All the variables which are caused by primary source I_0 will have an index of 0. (vector magnetic potential) and (magnetic field intensity) will be calculated according to the Biot-Savart law. Since the current in a conductor only flows in the direction of the $+x$ axis, the vector magnetic potential at the observation point on the z-axis will only have an x component, while the magnetic field intensity will only have a y and z component.

Vector magnetic potential A0 and magnetic field intensity at any point (x,y,z) are calculated as follows:

$$\dot{A} = \dot{A}_{0x} \cdot \boldsymbol{a}_x = \frac{\mu_0 \dot{I}_0}{4\pi} \ln \left(\frac{x + \frac{l}{2} + \sqrt{\left(x + \frac{l}{2}\right)^2 + y^2 + z^2}}{x - \frac{l}{2} + \sqrt{\left(x - \frac{l}{2}\right)^2 + y^2 + z^2}} \right) \cdot \boldsymbol{a}_x \tag{1}$$

$$\begin{aligned} \dot{\boldsymbol{H}}_0 &= \dot{H}_{0y} \cdot \boldsymbol{a}_y + \dot{H}_{0z} \cdot \boldsymbol{a}_z = \\ &= \frac{-z \cdot \dot{I}_0}{4\pi \cdot (y^2 + z^2)} \left(\frac{x + \frac{l}{2}}{\sqrt{\left(x + \frac{l}{2}\right)^2 + y^2 + z^2}} - \frac{x - \frac{l}{2}}{\sqrt{\left(x - \frac{l}{2}\right)^2 + y^2 + z^2}} \right) \cdot \boldsymbol{a}_y + \\ &+ \frac{y \cdot \dot{I}_0}{4\pi \cdot (y^2 + z^2)} \left(\frac{x + \frac{l}{2}}{\sqrt{\left(x + \frac{l}{2}\right)^2 + y^2 + z^2}} - \frac{x - \frac{l}{2}}{\sqrt{\left(x - \frac{l}{2}\right)^2 + y^2 + z^2}} \right) \cdot \boldsymbol{a}_z \end{aligned} \tag{2}$$

4 POSTAVLJANJE INTEGRALNIH JEDNADŽBI ZA MODEL

Neka se prema slikama 1 i 2 promatra navedeni model vodiča i feromagnetske ploče za kvazistatički slučaj.

Pretpostavlja se da su sve vremenske promjene sinusne s kružnom frekvencijom ω.

4 SETTING THE INTEGRAL EQUATIONS FOR THE MODEL

Let us consider the model of the current carrying conductor and ferromagnetic plate for the quasi-static case according to Figures 1 and 2.

It is assumed that all the time variations are sinusoidal with angular frequency ω:

$$\omega = 2 \cdot \pi \cdot f \tag{3}$$

Vanjski vremensko promjenjivi izvor $\dot{I}_0$ u feromagnetskoj ploči uzrokuje magnetizaciju i vrtložne struje što se modelira sljedećim sekundarnim izvorima [1] i [2]:

- $\dot{\boldsymbol{J}}_v$ – gustoća vrtložnih struja (A/m^2) koja se razvija unutar volumena V feromagnetske ploče,
- $\dot{\boldsymbol{K}}_m$ – linijska gustoća magnetizacijske struje (A/m) koja se javlja na površini S feromagnetske ploče (magnetizacijski strujni oblog),
- $\dot{\boldsymbol{\sigma}}_v$ – plošna gustoća električnog naboja (C/m^2) koja se pojavljuje na površini S feromagnetske ploče,

Prema [1] i [2], ukupno magnetsko polje kojeg stvaraju navedeni primarni i sekundarni izvori u bilo kojoj točki prostora je:

The outer time varying source $\dot{I}_0$ causes magnetic polarization and eddy currents in the ferromagnetic plate, which can be modeled by the following secondary sources [1] and [2]:

- $\dot{\boldsymbol{J}}_v$ – surface eddy current density (A/m^2), which appears inside the volume V of the ferromagnetic plate,
- $\dot{\boldsymbol{K}}_m$ – line magnetizing current density (A/m), which appears on the surface S of the ferromagnetic plate,
- $\dot{\boldsymbol{\sigma}}_v$ – surface electric charge density (C/m^2), which appears on the surface S of the ferromagnetic plate.

According to [1] and [2], the total magnetic field caused by the previously mentioned primary and secondary sources at any point of the space is as follows:

$$\dot{\boldsymbol{H}}(\boldsymbol{r}) = \dot{\boldsymbol{H}}_0(\boldsymbol{r}) + \dot{\boldsymbol{H}}_v(\boldsymbol{r}) + \dot{\boldsymbol{H}}_S(\boldsymbol{r}), \tag{4}$$

$$\dot{\boldsymbol{H}}(\boldsymbol{r}) = \dot{\boldsymbol{H}}_0(\boldsymbol{r}) + \frac{\mu_r}{4\pi}\int_V \dot{\boldsymbol{J}}_v(\boldsymbol{r}') \times \frac{\boldsymbol{R}}{R^3}\,\mathrm{d}V' + \frac{1}{4\pi}\int_S \dot{\boldsymbol{K}}_m(\boldsymbol{r}') \times \frac{\boldsymbol{R}}{R^3}\,\mathrm{d}S', \tag{5}$$

gdje je:

- $\dot{\boldsymbol{H}}_0(\boldsymbol{r})$ – magnetsko polje koje u točki $\boldsymbol{r}$ stvara primarni izvor $\dot{I}_0$,
- $\dot{\boldsymbol{H}}_v(\boldsymbol{r})$ – magnetsko polje koje u točki $\boldsymbol{r}$ stvara sekundarni izvor $\dot{\boldsymbol{J}}_v$,
- $\dot{\boldsymbol{H}}_s(\boldsymbol{r})$ – magnetsko polje koje u točki $\boldsymbol{r}$ stvara sekundarni izvor $\dot{\boldsymbol{K}}_m$,
- $\boldsymbol{r}$ – radij vektor promatrane točke (slika 3) u kojoj se računa polje (može biti bilo gdje u prostoru; unutar volumena V na površini S ili izvan volumena V),

where:

- $\dot{\boldsymbol{H}}_0(\boldsymbol{r})$ – the magnetic field at point $\boldsymbol{r}$ caused by the primary source $\dot{I}_0$,
- $\dot{\boldsymbol{H}}_v(\boldsymbol{r})$ – the magnetic field at point $\boldsymbol{r}$ caused by the secondary source $\dot{\boldsymbol{J}}_v$,
- $\dot{\boldsymbol{H}}_s(\boldsymbol{r})$ – the magnetic field at point caused by the secondary source $\dot{\boldsymbol{K}}_m$,
- $\boldsymbol{r}$ – the radius vector of the observation point (Figure 3), in which the field is calculated (can be anywhere in the space; inside the volume V, on the surface S or outside volume V),

r' – radij vektor tekuće točke integracije (slika 3) (može biti samo unutar volumena V ili na površini S),

$\dot{R}$ – vektor udaljenosti (slika 3).

r' – radius vector of the integration point (Figure 3) (can be either inside the volume V or on the surface S),

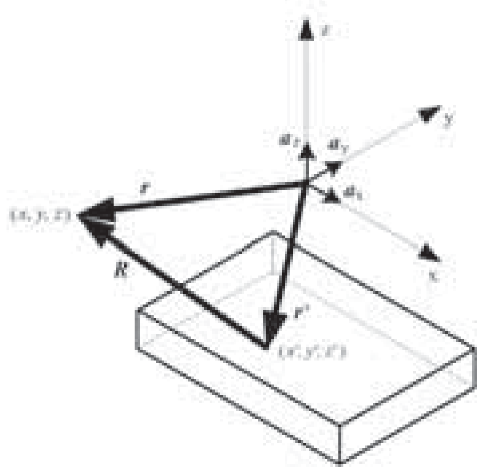
$\dot{R}$ – connecting vector (Figure 3).

$$\boldsymbol{r} = x \cdot \boldsymbol{a}_x + y \cdot \boldsymbol{a}_y + z \cdot \boldsymbol{a}_z \,, \tag{6}$$

$$\boldsymbol{r}' = x' \cdot \boldsymbol{a}_x + y' \cdot \boldsymbol{a}_y + z' \cdot \boldsymbol{a}_z \,, \tag{7}$$

$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' \,, \tag{8}$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \;. \tag{9}$$



Slika 3
Radij vektori
Figure 3
Radius vectors

Vektorski magnetski potencijal u bilo kojoj točki prostora dan je sljedećim izrazom [1] i [2]:

The vector magnetic potential at any point of the space can be calculated as follows [1] and [2]:

$$\dot{A}(\boldsymbol{r}) = \dot{A}_0(\boldsymbol{r}) + \dot{A}_V(\boldsymbol{r}) + \dot{A}_S(\boldsymbol{r}) \,, \tag{10}$$

$$\dot{A}(\boldsymbol{r}) = \dot{A}_0(\boldsymbol{r}) + \frac{\mu_0 \mu_r}{4\pi} \int_V \dot{J}_v(\boldsymbol{r}') \frac{\mathrm{d}V'}{R} + \frac{\mu_0}{4\pi} \int_S \dot{K}_m(\boldsymbol{r}') \frac{\mathrm{d}S'}{R} \,, \tag{11}$$

gdje je:

$\dot{A}_0(r)$ - vektorski magnetski potencijal kojeg u točki r stvara primarni izvor $\dot{I}_0$,
$\dot{A}_v(r)$ - vektorski magnetski potencijal kojeg u točki r stvara sekundarni izvor $\dot{J}_v$,
$\dot{A}_s(r)$ - vektorski magnetski potencijal kojeg u točki r stvara sekundarni izvor $\dot{K}_m$.

where:

$\dot{A}_0(r)$ – the vector magnetic potential at point r caused by the primary source $\dot{I}_0$,
$\dot{A}_v(r)$ – the vector magnetic potential at point r caused by the secondary source $\dot{J}_v$.
$\dot{A}_s(r)$ – the vector magnetic potential at point r caused by the secondary source $\dot{K}_m$.

Plošni naboj σ_v stvara skalarni električni potencijal φ koji je u bilo kojoj točki prostora dan izrazom [1] i [2]:

Surface charge σ_v creates electric scalar potential φ. At any point of the space, this potential can be calculated, as follows [1] and [2]:

$$\dot{\varphi}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_S \dot{\sigma}_v(\boldsymbol{r}')\frac{\mathrm{d}S'}{R}, \quad (12)$$

gdje je ε_0 relativna dielektrična konstanta u zraku: $\varepsilon_0 = 8{,}854 \cdot 10^{-12}$ F/m.

where ε_0 is the relative dielectric constant in the air $\varepsilon_0 = 8{,}854 \cdot 10^{-12}$ F/m.

Djelovanjem operatora grad na potencijal φ, za točku koja ne leži na S dobiva se:

If the operator grad is applied to the potential φ, S can be written as follows for any point that does not lie on the surface:

$$\mathrm{grad}\,\dot{\varphi}(\boldsymbol{r}) = -\frac{1}{4\pi\varepsilon_0}\int_S \dot{\sigma}_v(\boldsymbol{r}')\frac{\boldsymbol{R}}{R^3}\mathrm{d}S' \quad . \quad (13)$$

Za točku na graničnoj plohi S vrijedi (s unutrašnje strane površine S feromagnetskog materijala) [1]:

For a point on the surface S (from the inner side of the surface S of the ferromagnetic material), the following can be written [1]:

$$\mathrm{grad}\,\dot{\varphi}(\boldsymbol{r})\cdot\boldsymbol{n}(\boldsymbol{r}) = \frac{\dot{\sigma}_v}{2\varepsilon_0} - \frac{1}{4\pi\varepsilon_0}\int_S \dot{\sigma}_v(\boldsymbol{r}')\frac{\boldsymbol{R}\cdot\boldsymbol{n}(\boldsymbol{r})}{R^3}\mathrm{d}S'. \quad (14)$$

U bilo kojoj točki r unutar volumena V vrijedi [3]:

At any point r inside the volume V, the following can be written:

$$\dot{\boldsymbol{J}}_v(\boldsymbol{r}) = \gamma\cdot\dot{\boldsymbol{E}}(\boldsymbol{r}) = \gamma\cdot\left(-\mathrm{grad}\,\dot{\varphi}(\boldsymbol{r}) - \mathrm{j}\omega\dot{\boldsymbol{A}}(\boldsymbol{r})\right). \quad (15)$$

Uvrštavanjem izraza (11) i (13) u (15) dobiva se:

Inserting expressions (11) and (13) into (15), it follows:

$$\dot{\boldsymbol{J}}_v(\boldsymbol{r}) + \frac{j\omega\gamma\mu_0\mu_r}{4\pi} \int\limits_{\substack{V \\ \boldsymbol{r}' \neq \boldsymbol{r}}} \dot{\boldsymbol{J}}_v(\boldsymbol{r}') \frac{dV'}{R} + \frac{j\omega\gamma\mu_0}{4\pi} \int\limits_S \dot{\boldsymbol{K}}_m(\boldsymbol{r}') \frac{dS'}{R} - \frac{\gamma}{4\pi\varepsilon_0} \int\limits_S \dot{\sigma}_v(\boldsymbol{r}') \frac{\boldsymbol{R}}{R^3} dS' = -j\omega\gamma \dot{\boldsymbol{A}}_0(\boldsymbol{r}) \,. \tag{16}$$

U bilo kojoj točki $\boldsymbol{r}$ na površini S vrijedi [1]:

At any point $\boldsymbol{r}$ on the surface S, the following can be written [1]:

$$\dot{\boldsymbol{K}}_m(\boldsymbol{r}) = 2 \cdot \lambda_m \cdot \dot{\boldsymbol{H}}(\boldsymbol{r}) \times \boldsymbol{n}(\boldsymbol{r}) = 2 \cdot \lambda_m \cdot \left[\dot{\boldsymbol{H}}_0(\boldsymbol{r}) + \dot{\boldsymbol{H}}_v(\boldsymbol{r}) + \dot{\boldsymbol{H}}_S(\boldsymbol{r}) \right] \times \boldsymbol{n}(\boldsymbol{r}) \tag{17}$$

gdje je:

$\boldsymbol{n}(\boldsymbol{r})$ - okomica na površinu u točki $\boldsymbol{r}$.

Usmjerena je iz feromagnetskog materijala prema van.

where:

$\boldsymbol{n}(\boldsymbol{r})$ – Normal at the point $\boldsymbol{r}$.

The direction is from the ferromagnetic material towards the outside:

$$\lambda_m = \frac{\mu_r - 1}{\mu_r + 1} \,. \tag{18}$$

Uvrštavanjem izraza (5) u (17), dobiva se:

Inserting expression (5) into (17), it follows:

$$\dot{\boldsymbol{K}}_m(\boldsymbol{r}) - \frac{\lambda_m \cdot \mu_r}{2\pi} \int\limits_V \left(\dot{\boldsymbol{J}}_v(\boldsymbol{r}') \times \frac{\boldsymbol{R}}{R^3} \right) \times \boldsymbol{n}(\boldsymbol{r}) dV' - \frac{\lambda_m}{2\pi} \int\limits_{\substack{S \\ \boldsymbol{r}' \neq \boldsymbol{r}}} \left(\dot{\boldsymbol{K}}_m(\boldsymbol{r}') \times \frac{\boldsymbol{R}}{R^3} \right) \times \boldsymbol{n}(\boldsymbol{r}) dS' = 2 \cdot \lambda_m \cdot \dot{\boldsymbol{H}}_0(\boldsymbol{r}) \times \boldsymbol{n}(\boldsymbol{r}). \tag{19}$$

Također se za bilo koju točku $\boldsymbol{r}$ na površini S može napisati [3]:

At any point $\boldsymbol{r}$ on the surface S, the following can also be written [3]:

$$\dot{\boldsymbol{E}}(\boldsymbol{r}) = -\operatorname{grad} \dot{\varphi}(\boldsymbol{r}) - j\omega \dot{\boldsymbol{A}}(\boldsymbol{r}). \tag{20}$$

Iz uvjeta na granici feromagnetski materijal-zrak dobiva se [1]:

From the conditions at the boundary between the ferromagnetic material and the air, it follows [1]:

$$\dot{\boldsymbol{E}}(\boldsymbol{r}) \cdot \boldsymbol{n}(\boldsymbol{r}) = \dot{0}\,. \tag{21}$$

Množenjem (20) sa $\boldsymbol{n}(\boldsymbol{r})$ i uvrštavanjem (11), (14) i (21) u (20), slijedi:

Multiplying (20) by $\boldsymbol{n}(\boldsymbol{r})$ and inserting (11), (14) and (21) into (20), it follows:

$$\dot{\sigma}_{\mathrm{v}}(\boldsymbol{r}) + \frac{\mathrm{j}\omega\mu_0\mu_r\varepsilon_0}{2\pi}\int_V \dot{\boldsymbol{J}}_{\mathrm{v}}(\boldsymbol{r}')\cdot\frac{\boldsymbol{n}(\boldsymbol{r})}{R}\mathrm{d}V' + \frac{\mathrm{j}\omega\mu_0\varepsilon_0}{2\pi}\int_S \dot{\boldsymbol{K}}_{\mathrm{m}}(\boldsymbol{r}')\cdot\frac{\boldsymbol{n}(\boldsymbol{r})}{R}\mathrm{d}S' - \frac{1}{2\pi}\int_{\substack{S \\ r'\neq r}} \dot{\sigma}_{\mathrm{v}}(\boldsymbol{r}')\frac{\boldsymbol{R}\cdot\boldsymbol{n}(\boldsymbol{r})}{R^3}\mathrm{d}S'$$
$$= -2\,\mathrm{j}\omega\varepsilon_0\,\dot{\boldsymbol{A}}_0(\boldsymbol{r})\cdot\boldsymbol{n}(\boldsymbol{r}). \tag{22}$$

Izrazi (16), (19) i (22) su integralne jednadžbe iz kojih se dobivaju traženi sekundarni izvori.

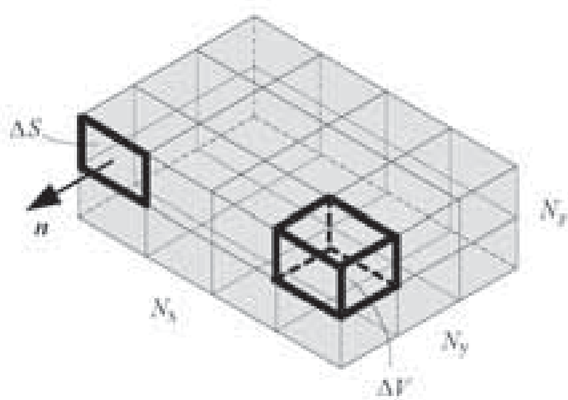
Expressions (16), (19) and (22) are integral equations from which secondary sources are calculated.

5 DISKRETIZACIJA INTEGRALNIH JEDNADŽBI

Da bi se odredili $\dot{\boldsymbol{J}}_{\mathrm{v}}$ (unutar volumena V), $\dot{\boldsymbol{K}}_{\mathrm{m}}$ i $\dot{\sigma}_{\mathrm{v}}$ (na površini S feromagnetske ploče), volumen V se dijeli N_{V} na volumnih elemenata, a površina S (oplošje kvadra) na N_{S} površinskih elemenata (slika 4). N_{X} je broj podjela u smjeru x osi, N_{Y} broj podjela u smjeru y osi i N_{Z} broj podjela u smjeru z osi. Kao elementarni volumen ΔV uzet će se kvadar zbog konstantnih granica integracije. Elementarna površina ΔS je pravokutnik.

5 DISCRETIZATION OF INTEGRAL EQUATIONS

In order to find $\dot{\boldsymbol{J}}_{\mathrm{v}}$ (inside the volume V), $\dot{\boldsymbol{K}}_{\mathrm{m}}$ and $\dot{\sigma}_{\mathrm{v}}$ (on the surface S of the ferromagnetic plate), volume V is divided into N_{V} volume elements, while the surface S (surface of the cuboid) is divided into N_{S} surface elements (Figure 4). N_{X} is the number of divisions in the direction of the x-axis, N_{Y} is number of divisions in the direction of the y-axis and N_{Z} is the number of divisions in the direction of the z-axis. Elementary volume ΔV is cuboid because of the constant borders of integration. Elementary surface ΔS is rectangular.



Slika 4
Podjela volumena V i površine S feromagnetske ploče na elementarne volumene ΔV i elementarne površine ΔS

Figure 4
Division of the volume V and surface S of the ferromagnetic plate into elementary volumes ΔV and elementary surfaces ΔS.

Prema slici 4 vrijedi:

According to Figure 4, it follows:

$$N_V = N_x \cdot N_y \cdot N_z , \tag{23}$$

$$N_S = 2 \cdot \left(N_x \cdot N_y + N_x \cdot N_z + N_y \cdot N_z \right). \tag{24}$$

Pretpostavit će se da je $\dot{\boldsymbol{J}}_v$ konstantan unutar svakog elementarnog volumena ΔV te da su $\dot{\boldsymbol{K}}_m$ i $\dot{\sigma}_v$ konstantni na svakoj elementarnoj površini ΔS.

Integralne jednadžbe (16), (19) i (22) napisat će se u obliku skalarnih jednadžbi po komponentama. Jednadžbe će se postavljati u težištima volumnih i plošnih elemenata.

Za bilo koju točku $\boldsymbol{r}_i$ težišta elementarnog volumena ΔV_i vrijedi ($\boldsymbol{r}_i'$ je težište tekućeg volumnog ili površinskog elementa po kojem se integrira):

It will be assumed that $\dot{\boldsymbol{J}}_v$ is constant inside every elementary volume ΔV and that $\dot{\boldsymbol{K}}_m$ and $\dot{\sigma}_v$ are constant on every elementary surface ΔS.

Integral equations (16), (19) and (22) will be written in the form of scalar equations for the components. The equations will be set in the center points of the volume and surface elements.

For any center point $\boldsymbol{r}_i$ of elementary volume ΔV_i the following can be written ($\boldsymbol{r}_i'$ is the center point of either volume or surface element being integrated):

$$\dot{J}_{vx}(\boldsymbol{r}_i) + \frac{\mathrm{j}\omega\gamma\mu_0\mu_r}{4\pi} \sum_{\substack{i=1 \\ \boldsymbol{r}_i' \neq \boldsymbol{r}_i}}^{N_V} \dot{J}_{vx}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{\mathrm{d}V'}{R} + \frac{\mathrm{j}\omega\gamma\mu_0}{4\pi} \sum_{i=1}^{N_S} \dot{K}_{mx}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{\mathrm{d}S'}{R} - \frac{\gamma}{4\pi\varepsilon_0} \sum_{i=1}^{N_S} \dot{\sigma}_v(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{x-x'}{R^3} \mathrm{d}S' = -\mathrm{j}\omega\gamma \dot{A}_{0x}(\boldsymbol{r}_i), \tag{25}$$

$$\dot{J}_{vy}(\boldsymbol{r}_i) + \frac{\mathrm{j}\omega\gamma\mu_0\mu_r}{4\pi} \sum_{\substack{i=1 \\ \boldsymbol{r}_i' \neq \boldsymbol{r}_i}}^{N_V} \dot{J}_{vy}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{\mathrm{d}V'}{R} + \frac{\mathrm{j}\omega\gamma\mu_0}{4\pi} \sum_{i=1}^{N_S} \dot{K}_{my}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{\mathrm{d}S'}{R} - \frac{\gamma}{4\pi\varepsilon_0} \sum_{i=1}^{N_S} \dot{\sigma}_v(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{y-y'}{R^3} \mathrm{d}S' = -\mathrm{j}\omega\gamma \dot{A}_{0y}(\boldsymbol{r}_i), \tag{26}$$

$$\dot{J}_{vz}(\boldsymbol{r}_i) + \frac{\mathrm{j}\omega\gamma\mu_0\mu_r}{4\pi} \sum_{\substack{i=1 \\ \boldsymbol{r}_i' \neq \boldsymbol{r}_i}}^{N_V} \dot{J}_{vz}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{\mathrm{d}V'}{R} + \frac{\mathrm{j}\omega\gamma\mu_0}{4\pi} \sum_{i=1}^{N_S} \dot{K}_{mz}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{\mathrm{d}S'}{R} - \frac{\gamma}{4\pi\varepsilon_0} \sum_{i=1}^{N_S} \dot{\sigma}_v(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{z-z'}{R^3} \mathrm{d}S' = -\mathrm{j}\omega\gamma \dot{A}_{0z}(\boldsymbol{r}_i). \tag{27}$$

Za bilo koju točku $\boldsymbol{r}_i$ težišta elementarne površine ΔS_i vrijedi:

For any center point $\boldsymbol{r}_i$ of elementary surface ΔS_i the following can be written:

$$\begin{aligned}
&\dot{K}_{mx}(\boldsymbol{r}_i)+\frac{\lambda_m \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vx}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(y-y')\cdot n_y(\boldsymbol{r}_i)+(z-z')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\\
&\frac{\lambda \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vy}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(x-x')\cdot n_y(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\frac{\lambda_m \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vz}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(x-x')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'+\\
&\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{mx}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(y-y')\cdot n_y(\boldsymbol{r}_i)+(z-z')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'-\\
&\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{my}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(x-x')\cdot n_y(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'-\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{mz}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(x-x')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'\\
&=2\cdot\lambda_m\cdot\left(\dot{H}_{0y}(\boldsymbol{r}_i)\cdot n_z(\boldsymbol{r}_i)-\dot{H}_{0z}(\boldsymbol{r}_i)\cdot n_y(\boldsymbol{r}_i)\right),
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\dot{K}_{my}(\boldsymbol{r})-\frac{\lambda_m \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vx}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(y-y')\cdot n_x(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'+\\
&\frac{\lambda_m \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vy}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(x-x')\cdot n_x(\boldsymbol{r}_i)+(z-z')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\\
&\frac{\lambda_m \cdot \mu_r}{2\pi}\sum_{i=1}^{N_V}\dot{J}_{vz}(\boldsymbol{r}_i')\int_{\Delta V_i}\frac{(y-y')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\\
&\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{mx}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(y-y')\cdot n_x(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'+\\
&\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{my}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(x-x')\cdot n_x(\boldsymbol{r}_i)+(z-z')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'-\\
&\frac{\lambda_m}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq \boldsymbol{r}_i}}^{N_S}\dot{K}_{mz}(\boldsymbol{r}_i')\int_{\Delta S_i}\frac{(y-y')\cdot n_z(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'\\
&=2\cdot\lambda_m\cdot\left(-\dot{H}_{0x}(\boldsymbol{r}_i)\cdot n_z(\boldsymbol{r}_i)+\dot{H}_{0z}(\boldsymbol{r}_i)\cdot n_x(\boldsymbol{r}_i)\right),
\end{aligned} \tag{29}$$

$$
\begin{aligned}
&\dot{K}_{\mathrm{mz}}(\boldsymbol{r}_i)-\frac{\lambda_{\mathrm{m}}\cdot\mu_{\mathrm{r}}}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vx}}(\boldsymbol{r}_i')\int\limits_{\Delta V_i}\frac{(z-z')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\\
&\frac{\lambda_{\mathrm{m}}\cdot\mu_{\mathrm{r}}}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vy}}(\boldsymbol{r}_i')\int\limits_{\Delta V_i}\frac{(z-z')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'+\\
&\frac{\lambda_{\mathrm{m}}\cdot\mu_{\mathrm{r}}}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vz}}(\boldsymbol{r}_i')\int\limits_{\Delta V_i}\frac{(x-x')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)+(y-y')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}V'-\\
&\frac{\lambda_{\mathrm{m}}}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq\boldsymbol{r}_i}}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{mx}}(\boldsymbol{r}_i')\int\limits_{\Delta S_i}\frac{(z-z')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'-\\
&\frac{\lambda_{\mathrm{m}}}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq\boldsymbol{r}_i}}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{my}}(\boldsymbol{r}_i')\int\limits_{\Delta S_i}\frac{(z-z')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'+\\
&\frac{\lambda_{\mathrm{m}}}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq\boldsymbol{r}_i}}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{mz}}(\boldsymbol{r}_i')\int\limits_{\Delta S_i}\frac{(x-x')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)+(y-y')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'\\
&=2\cdot\lambda_{\mathrm{m}}\cdot\left(\dot{H}_{0\mathrm{x}}(\boldsymbol{r}_i)\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)-\dot{H}_{0\mathrm{y}}(\boldsymbol{r}_i)\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)\right),
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&\dot{\sigma}_{\mathrm{v}}(\boldsymbol{r}_i)+\frac{\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vx}}(\boldsymbol{r}_i')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)\int\limits_{\Delta V_i}\frac{\mathrm{d}V'}{R^3}+\\
&\frac{\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vy}}(\boldsymbol{r}_i')\cdot n_{\mathrm{y}}(\mathrm{r}_i)\int\limits_{\Delta V_i}\frac{\mathrm{d}V'}{R^3}+\frac{\mathrm{j}\omega\mu_0\mu_{\mathrm{r}}\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{V}}}\dot{J}_{\mathrm{vz}}(\boldsymbol{r}_i')\cdot n_{\mathrm{z}}(\boldsymbol{r}_i)\int\limits_{\Delta V_i}\frac{\mathrm{d}V'}{R^3}+\\
&\frac{\mathrm{j}\omega\mu_0\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{mx}}(\boldsymbol{r}_i')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)\int\limits_{\Delta S_i}\frac{\mathrm{d}S'}{R^3}+\frac{\mathrm{j}\omega\mu_0\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{my}}(\boldsymbol{r}_i')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)\int\limits_{\Delta S_i}\frac{\mathrm{d}S'}{R^3}+\\
&\frac{\mathrm{j}\omega\mu_0\varepsilon_0}{2\pi}\sum_{i=1}^{N_{\mathrm{S}}}\dot{K}_{\mathrm{mz}}(\boldsymbol{r}_i')\cdot n_{\mathrm{z}}(\boldsymbol{r}_i)\int\limits_{\Delta S_i}\frac{\mathrm{d}S'}{R^3}-\\
&\frac{1}{2\pi}\sum_{\substack{i=1\\ \boldsymbol{r}_i'\neq\boldsymbol{r}_i}}^{N_{\mathrm{S}}}\dot{\sigma}_{\mathrm{v}}(\boldsymbol{r}_i')\int\limits_{\Delta S_i}\frac{(x-x')\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)+(y-y')\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)+(z-z')\cdot n_{\mathrm{z}}(\boldsymbol{r}_i)}{R^3}\mathrm{d}S'\\
&=-2\,\mathrm{j}\omega\varepsilon_0\cdot\left(\dot{A}_{0\mathrm{x}}(\boldsymbol{r}_i)\cdot n_{\mathrm{x}}(\boldsymbol{r}_i)+\dot{A}_{0\mathrm{y}}(\boldsymbol{r}_i)\cdot n_{\mathrm{y}}(\boldsymbol{r}_i)+\dot{A}_{0\mathrm{z}}(\boldsymbol{r}_i)\cdot n_{\mathrm{z}}(\boldsymbol{r}_i)\right),
\end{aligned}
\tag{31}
$$

Budući da ukupno postoji $3N_V + 4N_S$ nepoznatih sekundarnih izvora u težištima volumnih i površinskih elemenata, potrebno je napisati isto toliko skalarnih integralnih jednadžbi. U tu se svrhu svaka od jednadžbi (25), (26) i (27) postavlja u svim težišnim točkama volumnih elemenata (ukupno $3N_V$ skalarnih jednadžbi), dok se svaka od jednadžbi (28), (29), (30) i (31) postavlja u svim težišnim točkama površinskih elemenata (ukupno $4N_S$ skalarnih jednadžbi).

Since there are $3N_V + 4N_S$ of unknown secondary sources in the center points of the volume and surface elements, it is necessary to write the same number of scalar integral equations. Therefore, equations (25), (26) and (27) are set in all the center points of the volume elements (a total of $3N_V$ scalar equations), while equations (28), (29), (30) and (31) are set in all the center points of the surface elements (a total of $4N_S$ scalar equations).

Rješavanjem linearnog sustava od $3N_V + 4N_S$ jednadžbi sa $3N_V + 4N_S$ nepoznanica, dobivaju se traženi sekundarni izvori u težištima volumnih i površinskih elemenata.

By solving the linear system of $3N_V + 4N_S$ equations with $3N_V + 4N_S$ unknowns, the secondary sources at the center points of volume and surface elements are obtained.

Magnetsko polje u bilo kojoj točki $\boldsymbol{r}$ prostora je:

The magnetic field at any point $\boldsymbol{r}$ of the space is as follows:

$$\dot{H}_x(\boldsymbol{r}) = \dot{H}_{0x}(\boldsymbol{r}) + \frac{\mu_r}{4\pi}\left(\sum_{i=1}^{N_V} \dot{J}_{vy}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{z-z'}{R^3}\,\mathrm{d}V' - \sum_{i=1}^{N_V} \dot{J}_{vz}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{y-y'}{R^3}\,\mathrm{d}V' \right) + \frac{1}{4\pi}\left(\sum_{i=1}^{N_S} \dot{K}_{my}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{z-z'}{R^3}\,\mathrm{d}S' - \sum_{i=1}^{N_S} \dot{K}_{mz}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{y-y'}{R^3}\,\mathrm{d}S' \right), \tag{32}$$

$$\dot{H}_y(\boldsymbol{r}) = \dot{H}_{0y}(\boldsymbol{r}) + \frac{\mu_r}{4\pi}\left(-\sum_{i=1}^{N_V} \dot{J}_{vx}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{z-z'}{R^3}\,\mathrm{d}V' + \sum_{i=1}^{N_V} \dot{J}_{vz}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{x-x'}{R^3}\,\mathrm{d}V' \right) + \frac{1}{4\pi}\left(-\sum_{i=1}^{N_S} \dot{K}_{mx}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{z-z'}{R^3}\,\mathrm{d}S' + \sum_{i=1}^{N_S} \dot{K}_{mz}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{x-x'}{R^3}\,\mathrm{d}S' \right), \tag{33}$$

$$\dot{H}_z(\boldsymbol{r}) = \dot{H}_{0z}(\boldsymbol{r}) + \frac{\mu_r}{4\pi}\left(\sum_{i=1}^{N_V} \dot{J}_{vx}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{y-y'}{R^3}\,\mathrm{d}V' - \sum_{i=1}^{N_V} \dot{J}_{vy}(\boldsymbol{r}_i') \int_{\Delta V_i} \frac{x-x'}{R^3}\,\mathrm{d}V' \right) + \frac{1}{4\pi}\left(\sum_{i=1}^{N_S} \dot{K}_{mx}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{y-y'}{R^3}\,\mathrm{d}S' - \sum_{i=1}^{N_S} \dot{K}_{my}(\boldsymbol{r}_i') \int_{\Delta S_i} \frac{x-x'}{R^3}\,\mathrm{d}S' \right). \tag{34}$$

Za navedeni model u kojem se polje računa u smjeru osi z je $H_{0X} = H_{0Z} = 0$.

For the defined model, $H_{0X} = H_{0Z} = 0$.

Magnetska indukcija B se dobiva množenjem magnetskog polja sa μ_0:

Magnetic flux density B is obtained by multiplying the magnetic field intensity by μ_0:

$$\dot{B}_x(\boldsymbol{r}) = \mu_0 \cdot \dot{H}_x(\boldsymbol{r}), \tag{35}$$

$$\dot{B}_y(\boldsymbol{r}) = \mu_0 \cdot \dot{H}_y(\boldsymbol{r}), \tag{36}$$

$$\dot{B}_z(\boldsymbol{r}) = \mu_0 \cdot \dot{H}_z(\boldsymbol{r}). \tag{37}$$

Efektivna vrijednost magnetske indukcije u bilo kojoj točki prostora je:

The effective value of the magnetic flux density at any point of the space is as follows:

$$B(\boldsymbol{r}) = \sqrt{\left|\dot{B}_x(\boldsymbol{r})\right|^2 + \left|\dot{B}_y(\boldsymbol{r})\right|^2 + \left|\dot{B}_z(\boldsymbol{r})\right|^2}. \tag{38}$$

6 PRIKAZ REZULTATA MJERENJA

Mjerenje magnetskog polja obavljeno je na pet modela prema slici 1. Podaci o modelima nalaze se u tablici 1.

6 MEASUREMENT RESULTS

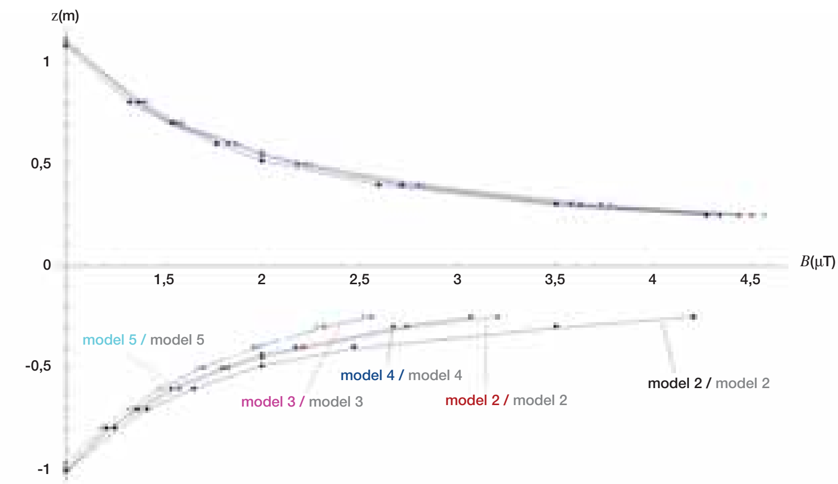
Magnetic field measurement was performed on five models according to Figure 1. Model data are given in Table 1.

Tablica 1 – Podaci modela
Table 1 – Model data

MODEL / Model	I_0 (A)	f (Hz)	l (m)	p (m)	d (m)	a (m)	b (m)	c (m)	z_p (m)
1	5,26	50	7,3	0,175	0,005	-	-	-	-
2	5,26	50	7,3	0,175	0,005	0,51	0,80	0,00125	-0,146
3	5,26	50	7,3	0,175	0,005	0,51	0,80	0,00250	-0,146
4	5,26	50	7,3	0,175	0,005	0,80	0,51	0,00125	-0,146
5	5,26	50	7,3	0,175	0,005	0,80	0,51	0,00250	-0,146

Krivulje izmjerenih vrijednosti magnetske indukcije po z osi na modelima prikazane su na slici 5. Vidljivo je da se umetanjem feromagnetske ploče ispod vodiča smanjuje magnetsko polje ispod ploče, dok se magnetsko polje iznad vodiča neznatno povećava. Što je ploča deblja, prigušenje polja ispod ploče je veće.

The curves of the measured values of the magnetic flux density along the z-axis are given in Figure 5. It can be seen that the inserted ferromagnetic plate beneath the conductor decreases the magnetic field under the plate and slightly increases the magnetic field above the conductor. The thicker the plate, the greater the attenuation of the field under the plate.



Slika 5
Krivulje izmjerenih vrijednosti B po z osi na modelima
Figure 5
Curves of the measured values of B along the z-axis for the models

7 USPOREDBA REZULTATA PRORAČUNA I MJERENJA

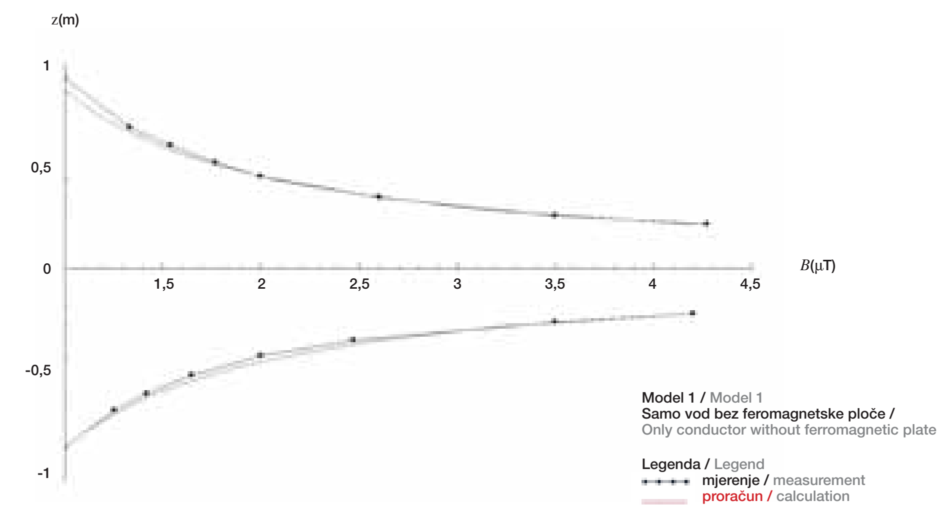
Kao što je već rečeno, proračun magnetskog polja na modelu napravljen uz zanemarenje progiba i promjera vodiča te zanemarenje utjecaja svih ostalih izvora i materijala na polje vodiča (osim navedene feromagnetske ploče). Krivulje usporedbe mjerenih i računskih vrijednosti B za modele 1 do 5 prikazane su na slikama 6 do 10. Uspoređene krivulje dobro se slažu.

Odstupanja od oko 15 % postoje u najvišim mjernim točkama iznad vodiča (cca na 1 m iznad vodiča) zbog utjecaja polja ostalih izvora.

7 COMPARISON OF THE CALCULATED AND MEASURED DATA

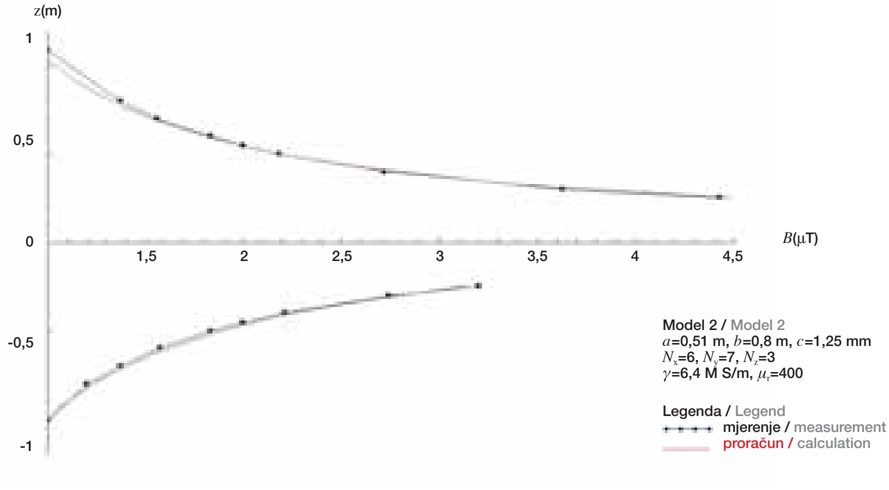
As previously stated, the deflection and diameter of the conductor are ignored in the calculation of the magnetic field intensity of the model. Furthermore, the influence of other sources and materials (except the ferromagnetic plate) on the magnetic field of the conductor is ignored. The measured and calculated curves of B for models 1 to 5 are shown in Figures 6 to 10. The compared curves have good matching.

Deviation of approximately 15 % exists only in the highest measured points above the conductor (approximately 1 m above the conductor) due to the influence of the other conductors.

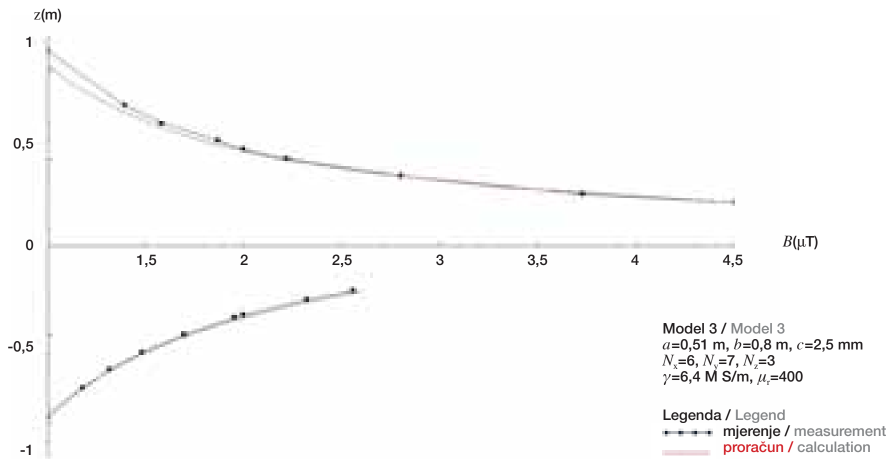


Slika 6
Usporedba krivulja izmjerenih i računskih vrijednosti B za model 1
Figure 6
Comparison of the measured and calculated curves of B for Model 1

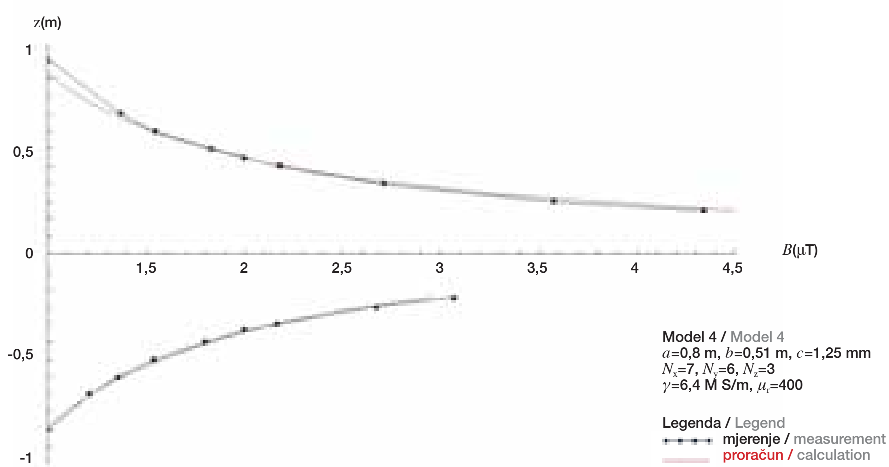
Slika 7
Usporedba krivulja izmjerenih i računskih vrijednosti B za model 2
Figure 7
Comparison of the measured and calculated curves of B for Model 2

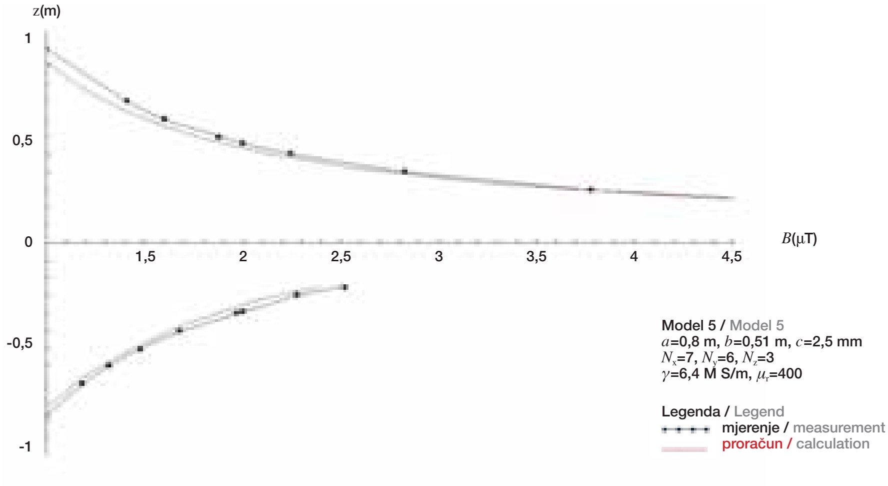


Slika 8
Usporedba krivulja izmjerenih i računskih vrijednosti B za model 3
Figure 8
Comparison of the measured and calculated curves of B for Model 3



Slika 9
Usporedba krivulja izmjerenih i računskih vrijednosti B za model 4
Figure 9
Comparison of the measured and calculated curves of B for Model 4





Slika 10
Usporedba krivulja izmjerenih i računskih vrijednosti B za model 5

Figure 10
Comparison of the measured and calculated curves of B for Model 5

8 ZAKLJUČAK

3D proračun kvazistatičkog magnetskog polja preko integralnih jednadžbi pokazuje se vrlo efikasnim u prostoru u kojem se traži polje daleko od izvora i u kojem nema puno različitih feromagnetskih materijala. Pri tome se pretpostavlja da su vodljivost i permeabilnost feromagnetskog materijala konstantni.

Za navedeni model vodiča i feromagnetske ploče postoji vrlo dobro slaganje rezultata mjerenja i proračuna. Feromagnetska ploča, umetnuta ispod vodiča, smanjuje magnetsko polje vodiča u području ispod feromagnetske ploče, a neznatno povećava magnetsko polje iznad vodiča.

8 CONCLUSION

The 3D calculation of a quasistatic magnetic field by means of integral equations is very effective for a space in which the field to be calculated is far from the source and in which there are not many different ferromagnetic materials. The conductivity and permeability of the ferromagnetic material are assumed to be constant.

For the defined model of the current carrying conductor and ferromagnetic plate, the results of the measurements and calculations are in good agreement. The insertion of a ferromagnetic plate beneath under the conductor decreases the magnetic field under the plate and slightly increases the magnetic field above the conductor.

LITERATURA / REFERENCES

[1] MATJAN, J., Rješavanje zadaća elektromagnetskog polja integralnim jednadžbama, doktorska disertacija, ETF, Zagreb, 1977.
[2] TOZONI, O. V., I. D. MAYERGOIZ, Calculation of three-dimensional Electromagnetic Fields, Tekhnika, Kiev, 1974
[3] HAZNADAR, Z. i ŠTIH, Ž., Elektromagnetizam 1, Školska knjiga, Zagreb, 1997.
[4] HAZNADAR, Z. i ŠTIH, Ž., Elektromagnetizam 2, Školska knjiga, Zagreb, 1997.

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