

# PRIMJENA MÖBIUSOVE VRPCE U ELEKTROTEHNICI APPLICATION OF THE MÖBIUS STRIP IN ELECTRICAL ENGINEERING

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Jednostrana dvodimenzionalna površina nazvana Möbiusovom vrpcom osim u umjetnosti i ostalim granama tehnike rabi se i u elektrotehnici. Najčešće za izradu nisko omskih neinduktivnih otpornika, posebice u visokofrekvencijskim i impulsnim uređajima, zatim sustava kondenzatora u visokofrekvencijskim sklopovima, mikrovalnih rezonatora i filtara.

O tomu postoje četiri patenta prijavljena u SAD-u.

The one-sided two-dimensional surface known as the Möbius strip, in addition to applications in the arts and various branches of technology, is also used in electrical engineering, most frequently in the construction of low-ohm non-inductive resistors, particularly in high frequency and pulse devices, as well as capacitor systems in high frequency units and microwave resonators and filters, for which there are four registered patents in the United States.

**Ključne riječi: kondenzator, neinduktivni otpornik, rezonator**  
**Key words: capacitor, noninductive resistor, resonator**



## 1 UVOD

Njemački astronom i matematičar August Ferdinand Möbius (1790. – 1868.) uz Georga Friedricha Bernharda Riemanna (1826. – 1866.) smatra se jednim od njemačkih pionira neeuclidске geometrije i topologije. Topologija (grčki: *topos* – mjesto, *logos* – proučavanje) je, s više grana, dio čiste matematike, zapravo moderna geometrija, koja se bavi svojstvima objekata koja su sačuvana pri njihovoj deformaciji, uvrtanju i rastezanju, dok rezanje ili deranje nije dopušteno.

Tako je npr. kružnica topološki jednaka elipsi, a kugla elipsoidu, jer su potonji nastali rastezanjem kružnice odnosno kugle. Möbius je 1858. godine otkrio, a 1865. godine obznanio značajke jednostrane dvodimenzionalne površine nazvane po njemu Möbiusovom vrpcom (*Möbius strip*, *Möbius band*).

Te značajke je istovremeno, a po nekim autorima i nekoliko godina ranije, neovisno o Möbiusu, otkrio i njemački svestrani znanstvenik Johann Benedict Listing (1808. – 1882.), ali ga se u literaturi, s tim u svezi, rijetko spominje. Listing je, među ostalim, prvi upotrijebio izričaje topologija i mikron.

Sva trojica spomenutih znanstvenika bili su učenici i/ili suradnici velikog njemačkog matematičara, astronoma, geodeta, fizičara, topografa itd. Johanna Friedricha Carla Gaussa (1777. – 1855.). Na tog velikana podsjeća nas više desetaka nazivlja s njegovim prezimenom iz područja matematike, fizike, astronomije itd. Spomenimo neka: zakon u elektrotehnici, teorem o divergenciji u vektorskoj analizi, stara jedinica za magnetsku indukciju, razdioba odnosno krivulja vjerojatnosti i krater na Mjesecu. Njegov se lik višekратно pojavljivao na poštanskim markama, nalazio se na novčanici od 10 DM itd.

Nije rijetkost da mnogi pronalasci na jednom području ljudske djelatnosti nakon više desetljeća ili čak stoljeća daju ideje za druge, na njima temeljenim, pronalascima iz sasvim drugih područja. To je slučaj i s Möbiusovom vrpcom, koja je prvotno bila zanimljiva samo matematičarima, a ovdje je opisana njezina primjena u elektrotehnici.

## 1 INTRODUCTION

The German astronomer and mathematician August Ferdinand Möbius (1790–1868) is considered one of the German pioneers of non-Euclidean geometry and topology, together with Georg Friedrich Bernhard Riemann (1826–1866). Topology (Greek: *topos* – place, *logos* – study) is, with several branches, a part of pure mathematics, actually modern geometry, engaged in the properties of objects that are retained during their deformation, twisting and stretching, while cutting or tearing are not permitted.

Thus, for example, a circle is topologically equal to an ellipse and a sphere is equal to an ellipsoid, since an ellipse occurs with the stretching of a circle and an ellipsoid with the stretching of a sphere. In 1858, Möbius discovered and in 1865 published the characteristics of a one-sided two-dimensional surface named after him, the Möbius strip or Möbius band.

According to some authors, at the same time or several years earlier, independently of Möbius, these characteristics had been discovered by the German all-around scientist Johann Benedict Listing (1808–1882), who is, however, less frequently mentioned in this context in the literature. Listing, among other things, was the first to use the terms topology and micron.

All three of the scientists mentioned were either students or associates of the great German mathematician, astronomer, geodesist, physicist and topographer Johann Friedrich Carl Gauss (1777–1855). We are reminded of this great man by more than ten terms using his surname from the fields of mathematics, physics, astronomy etc. We mention Gauss' law in electrical engineering, Gauss' theorem of divergence in vector analysis, the old Gauss unit for magnetic induction, Gauss distribution, i.e. the Gaussian probability distribution curve and a crater on the moon. His portrait has appeared many times on postage stamps and could be found on 10 DM bills etc.

It is not uncommon for many discoveries in one area of human activity to provide ideas for discoveries in other areas after many decades or even centuries. This is the case with the Möbius strip, which was initially only of interest to mathematicians but also has applications in electrical engineering, as described in this article.

## 2 MÖBIUSOVA VRPCA

Ako se jedan kraj, npr. pravokutne papirnatih vrpce, zakrene uzduž dulje osi za pola okretaja, tj. za  $180^\circ$  i spoji s drugim krajem dobije se poseban oblik, s **jednom stranom i rubom**, tzv. Möbiusova vrpca ili petlja (slika 1). Da ima samo jednu stranu lako je dokazati bilo kojim pisalom. Ako se iz jedne točke, sredinom vrpce, pisalom započne crtati linija vratit će se u istu točku bez prijelaza preko ruba vrpce. Sličnim postupkom, npr. označavanjem markerom, može se dokazati da taj oblik ima samo jedan rub. Zanimljivo je, što će se dobiti ako se škarama reže uzduž središnje linije tog oblika, ili linijom povučenom trećinom širine vrpce.

Möbiusova vrpca ima široku primjenu. U starim industrijskim objektima snaga jednog pogonskog stroja prenosila se na više radnih strojeva kožnatim ili gumenim remenima. Ako je remen bio u obliku Möbiusove vrpce jednoliko su se trošile obje strane, dok se s običnim remenom trošila samo jedna njegova strana. Isto se tako magnetofonska vrpca u obliku Möbiusove vrpce rabila u uređajima za neprekinuto snimanje, jer se time postizalo dvostruko vrijeme registracije, odnosno reprodukcije.

## 2 THE MÖBIUS STRIP

If one end of a strip of paper is twisted along its length by a half turn, i.e. by  $180$  degrees, and connected to the other end, a special shape is obtained, with **one side and one edge**, the so-called Möbius strip or band (Figure 1). It is easy to demonstrate that it has only one side with any writing implement. If from one point in the center of the strip one starts to draw a line, it will return to the same point without crossing the edge of the strip. Through a similar approach, for example marking with a marker, it can be demonstrated that this form has only one edge. It is interesting what will be obtained if one cuts along the center of the line of this form, or along the line at a third of the width of the strip.

The Möbius strip has wide applications. In old industrial facilities, the power from a drive machine was transmitted to several other machines with leather or rubber belts. If the belt was in the shape of a Möbius strip, it would wear out evenly, while an ordinary belt would wear out on only one of its sides. Similarly, recording tapes in the shape of a Möbius strip are used in devices for uninterrupted recording because they provide double recording and playing times.



**Slika 1**  
Möbiusova vrpca  
(petlja)  
**Figure 1**  
Möbius strip  
(band)

Zakrene li se jedan kraj vrpce za dva poluokreta, tj. umjesto  $180^\circ$  za  $360^\circ$ , dobije se dvostrana dvorubna vrpca. Dakle, Möbiusova vrpca postiže se samo s **neparnim brojem poluzakretaja** jednoga kraja.

Veliki je broj primjena Möbiusove vrpce u kiparstvu, grafici i ostalim granama umjetnosti, tehnici itd., a nalazi se i na brazilskoj poštanskoj marki. Zanimljivo je, da se u časopisu Nature od 23.5.2002. godine spominje, da su pronađeni kri-

If one end of a strip is twisted  $360^\circ$  by two turns, instead of  $180^\circ$ , a double two-sided strip is obtained. Thus, a Möbius strip is only obtained with **an odd number of half twists** of one end.

There are a large number of applications of the Möbius strip in sculpture, graphics and other branches of art, technology etc., and one is pictured on a Brazilian postage stamp. It is interesting that the May 23, 2002 issue of the journal Nature mentions that crystals have been found of

stali nekih kemijskih spojeva u obliku te vrpce, što svakako nema veze sa samim Möbiusom.

Najpoznatija primjena Möbiusove vrpce je međunarodni znak za reciklažu, u obliku trokuta s tri strjelice (slika 2), koji se nalazi na odgovarajućoj ambalaži. Veliki broj tekstova o Möbiusovoj vrpici mogu se naći na internetu [1] i [2].

Valja napomenuti da se prezime Möbius u tekstovima na engleskom jeziku piše dvojako, kao Mobius i Moebius.

some chemical compounds in the form of this strip, which certainly has no connection whatsoever with Möbius.

The best known application of the Möbius strip is the international symbol for recycling, in the form of a triangle with three arrows (Figure 2), used on packaging materials. A large number of texts on the Möbius strip can be found on the Internet, for example [1] and [2].

It should be mentioned that the surname of Möbius can also be written as Mobius and Moebius in English texts.

**Slika 2**  
Međunarodni znak  
za reciklažu  
Figure 2  
International  
recycling symbol



### 3 PRIMJENA MÖBIUSOVE VRPCE U ELEKTROTEHNICI

Koliko je poznato, do danas je u SAD-u patentirano više naprava, uređaja i elemenata koji se temelje na načelu Möbiusove vrpce, među kojima su i tri električna elementa. I Teslin patent za namot elektromagneta iz 1894. godine također se pripisuje načelu Möbiusove vrpce.

#### 3.1 Möbiusov neinduktivni otpornik

Elementi električnih uređaja otpornici, kondenzatori i svitci, posebno oni precizni, trebaju imati što manje parazitskih sastavnica (komponenata). Zbog struje kroz otpornik nastaje unutar i izvan njega magnetsko polje, pa stoga otpornik ima određeni samoinduktivitet  $L$ , kojeg se može zamisliti u seriji s otporom  $R$ . Taj induktivitet dolazi do izražaja već pri niskim frekvencijama. Između zavoja, kao i između različitih dijelova otpornika i susjednih metalnih predmeta postoji mnoštvo kapaciteta malih vrijednosti, koje se može nadomjestiti jednim

### 3 APPLICATION OF THE MÖBIUS STRIP IN ELECTRICAL ENGINEERING

Several devices and elements based upon the Möbius strip have been patented in the United States, including three electrical elements. Tesla's 1894 patent for a coil for electromagnets is also attributed to the principle of the Möbius strip.

#### 3.1 The Möbius noninductive resistor

The elements of electrical devices such as resistors, capacitors and coils, particularly those which are more precise, should have as few parasitic components as possible. Due to current passing through a resistor, a magnetic field is formed on the inside and outside, and thus the resistor has a certain self-inductance  $L$ , which we can imagine in a series with resistance  $R$ . This inductance is already evident at low frequencies. Between the turns, as well as between various parts of a resistor and neighboring metal objects, there are many low value capacitances, which can be substi-



kapacitetom  $C$  između početka i kraja otpornika. Vlastiti induktivitet i kapacitet otpornika izazvati će u strujnom krugu fazni pomak:

$$\varphi \approx \operatorname{tg} \varphi = \omega \left[ \frac{L}{R} (1 - \omega^2 LC) - RC \right] = \omega \tau \quad (1)$$

između struje i napona, pa će otpornik, posebice kod viših frekvencija, djelovati kao impedancija. Stoga se posebnim načinima izrade preciznih otpornika nastoji smanjiti vlastiti induktivitet i kapacitet, jer se oni ne mogu potpuno otkloniti. Međutim, međusobno se mogu tako uskladiti da cijeli otpornik djeluje u strujnom krugu kao djelatni otpor. Kakvoća otpornika za izmjeničnu struju, pri kružnoj frekvenciji  $\omega = 2\pi f$  s obzirom na parazitske sastavnice, iskazuje se vremenskom stalnicom (konstantom)  $\tau$  [3]:

$$\tau = \frac{L}{R} (1 - \omega^2 LC) - RC. \quad (2)$$

Za frekvencije do 20 kHz je  $\omega^2 LC \ll 1$ , pa se ovaj izraz može pojednostaviti u:

$$\tau \approx \frac{L}{R} - RC. \quad (3)$$

Ako se postigne da je  $L/R = RC$ , vremenska stalnica bit će jednaka ničtici, pa otpornik, unatoč samoinduktivitetu i parazitskom kapacitetu, djeluje kao djelatni otpor.

Bez posebnih mjera, kod otpornika velikih otpora prevladava utjecaj parazitskih kapaciteta, a kod malih otpora utjecaj induktiviteta. Stoga se pri izradi niskoomskih otpornika posebnim zahvatima, npr. načinima namatanja, nastoji smanjiti njihov samoinduktivitet. Žičani otpornici otpora reda veličine 100  $\Omega$  ili manji najčešće se izrađuju bifilarno ili namataju prema Ayrton-Perryu, a većeg otpora prema Chaperonu itd. [3]. Kvalitetni otpornici imaju vremensku stalnicu reda veličine 10 nanosekundi.

Richard L. Davies iz Sandia Laboratories (SAD) patentirao je 16.8.1966. godine [4] neinduktivni električni otpornik, pretežito nisko omski, posebice za primjenu u visokofrekvencijskim i impulsnim

tuted by the equivalent capacitance  $C$  between the beginning and the end of the resistor. The self inductance and capacitance of a resistor causes a phase shift between the current and the voltage in a circuit:

Thus, the resistor behaves as an impedance, especially at higher frequencies. Therefore, high-precision resistors are constructed in a specific manner in order to reduce self-inductance and capacitance, since they cannot be completely eliminated. However, they can be matched so that the whole resistor acts as an active resistor in a circuit. The quality of a resistor in an alternating current circuit in respect to its parasitic components at the angular frequency  $\omega = 2\pi f$  is expressed by the time constant  $\tau$  [1]:

For frequencies of up to 20 kHz,  $\omega^2 LC \ll 1$  and therefore this expression can be simplified as follows:

If  $L/R = RC$ , the time constant will be zero, so the resistor, despite self-inductance and parasitic capacitance, will act as an active resistor.

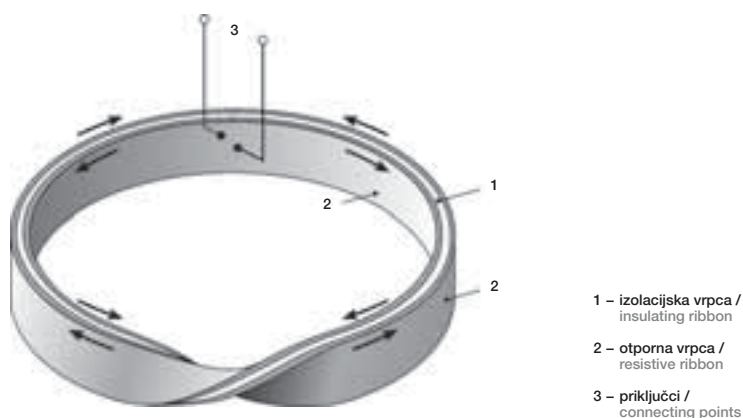
Without special measures, in high value resistors the impact of parasitic capacitances is dominant, while in low value resistors the inductive impact is dominant. Therefore, when devising resistors of low resistance value using specialized procedures, for example winding procedures, attempts are made to reduce their self-inductance. Wire resistors of 100  $\Omega$  or lower are most often made with bifilar windings according to Ayrton-Perry and of greater resistance according to Chaperon etc. [3]. High quality resistors have a time constant of an order of magnitude of 10 nanoseconds.

On August 16, 1966, Richard L. Davies of Sandia Laboratories (United States) [4] patented a noninductive electrical resistor, predominantly low ohmic, particularly for applications in high frequency and

elektroničkim sklopovima, npr. u tadašnjim radarima, koji su radili na frekvencijama do nekoliko gigaherca. Otpornik se sastoji od dvije vrpce prikladnog otpornog materijala, iste duljine i širine, učvršćene na suprotnim stranama jedne izolacijske vrpce. Jedni krajevi te kombinacije zakrenu se za 180° i spoje s drugim krajevima, tj. oblikuje se Möbiusova vrpca (slika 3). Krajevi otpornih vrpca spoje se lemljenjem. Umjesto vrpce može se rabiti izolirana otporna žica, npr. manganinska. Priključci na otporničke vrpce, što je od posebne važnosti, moraju biti točno jedan nasuprot drugome. U suprotnome otpornik ima induktivnu sastavnicu koja je najveća kada su priključci razmaknuti za polovicu duljine petlje. Struje, odnosno impulsi u sklopovima, od priključka teku otpornim vrpicama u suprotnim pravcima, tako da se njihova elektromagnetska polja poništavaju, pa se dobije neinduktivni otpornik vrlo male vremenske stalnice. Jedna i druga vrpca zapravo su paralelno spojene. Poprečni presjek otpornika pokazuje da je to kondenzator, pa postoji određena kapacitivna sastavnica.

pulse electronic circuits, for example in the radar installations of the time, which operated at frequencies of up to several GHz. The resistor consists of two ribbons of a suitable resistive material of the same length and width, affixed on opposite ends of a strip of dielectric. One end of this assembly is twisted 180° and is joined to the other end, i.e. a Möbius strip is formed (Figure 3). The ends of the resistive ribbons are soldered together. Instead of ribbons, it is possible to use resistive wire, e.g., Manganin. The connection points on resistive ribbons must be precisely one opposite the other, which is of particular importance. Otherwise, the resistor would have an inductive component which is greatest when the distance between the connection points is half the loop length. Currents or pulses flow through the resistive ribbons in opposite directions, so that their electromagnetic fields cancel each other. Thus, a non-inductive resistor with a very small time constant is obtained. Actually, the ribbons are connected in parallel. A cross-sectional view reveals that it is a capacitor, and there is a certain capacitive component.

**Slika 3**  
Möbiusov otpornik  
Figure 3  
Möbius resistor



Eksperimentalni primjerci takvih otpornika, otpora reda veličine 10 Ω, imali su induktivitete reda veličine 10 nH i kapacitete reda 0,1 pF, dakle vremenske stalnice reda veličine nanosekunde. Vrijeme porasta impulsa, tj. vrijeme potrebno da impuls od 10 % dostigne razinu od 90 % svoje konačne vrijednosti, kod 1 kV, bilo je oko 0,1 μs.

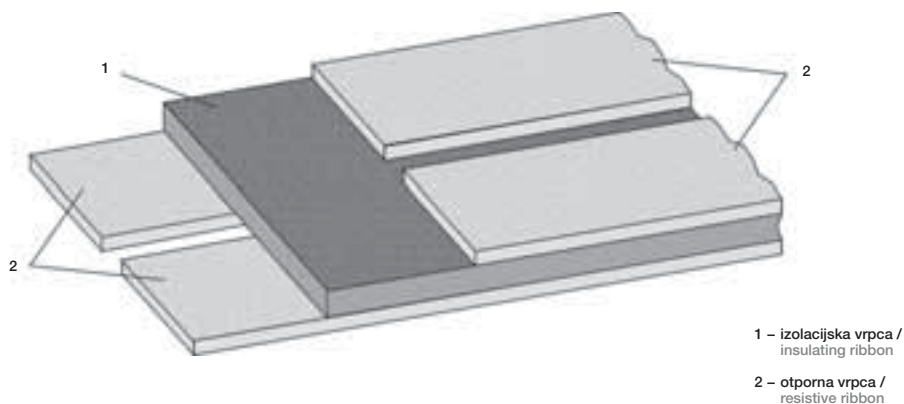
Na jednoj izolacijskoj vrpici mogu se načiniti dva ili više takvih otpornih elemenata, s međusobim razmacima od oko 2 mm, koji se mogu, po želji, spajati serijski ili paralelno (slika 4). Sastavnice takvog sustava nemaju međusobnog utjecaja, kao niti utjecaja bliskih metalnih objekata i vanjskih polja. Značajke Möbiusovog otpornika ne mijenjaju se njegovom duljinom ili oblikom. To znači da se

Experimental samples of such resistors, of an order of magnitude of 10 ohms, had inductances of an order of magnitude of 10 nH and capacitances of an order of magnitude of 0,1 pF, and thus a time constant of an order of magnitude of a nanosecond. The pulse rise time, i.e., the time necessary for a pulse to increase from 10 % to 90 % of its peak value, was approximately 0,1 μs at 1 kV.

One or more such resistive elements can be applied to an insulating ribbon with approximately 2 mm spacings. Resistive elements can be connected in series or in parallel (Figure 4). The components of such a system do not affect each other and do not couple electromagnetically to other metallic objects. The characteristics of a Möbius resistor do not change with

Möbiusov otpornik može omotati oko valjkastog tijela ili tanke kartice, pa čak ga oblikovati u kuglu.

its length or form. This means that a Möbius resistor can be wound around a cylindrical core or a thin card, and can even be formed into a ball.



**Slika 4**  
Möbiusov  
višestruki otpornik  
**Figure 4**  
Möbius combined  
resistor

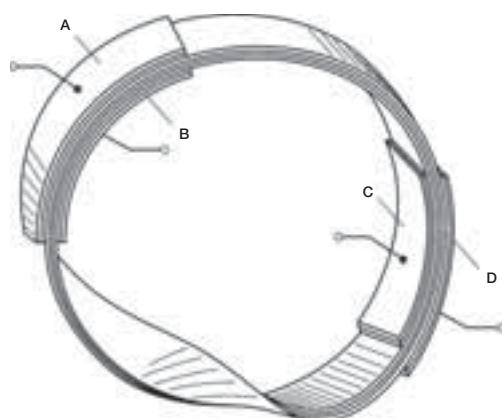
1 – izolacijska vrpca /  
insulating ribbon  
2 – otporna vrpca /  
resistive ribbon

### 3.2 Möbiusov kondenzator

Dvadeset godina nakon patenta za neinduktivni Möbiusov otpornik, na njegovoj je osnovi Thomas J. Brown patentirao 8.7.1986. godine Möbiusov kondenzator [5]. Jednostavno je na, prije spomenute otporne vrpce, ili općenito vrpce od vodljiva materijala Möbiusova otpornika, stavio izolacijske slojeve i na njima vodljive vrpce, tako da su te kombinacije kraće od vrpce samog otpornika i nalaze se jedna nasuprot druge (slika 5). Vodičima se na vanjskim vodljivim vrpčama (A,B,C i D) priključuju u električni krug. Mogu biti jedan ili dva para takvih kondenzatora uzduž opsega ovog složenog sustava. Oni imaju ukupno sedam vrpca, izolacijskih i vodljivih. Duljinom i širinom vrpca, te debljinom izolacije, mogu se ugađati vrijednosti kapaciteta.

### 3.2 Möbius capacitor

Twenty years after the patent was issued for the non-inductive Möbius resistor, Thomas J. Brown patented the Möbius capacitor on July 8, 1986 [5]. On top of the previously mentioned resistive ribbons or, generally, the ribbon conductors of a Möbius resistor, he simply layered a dielectric material and then layered this with ribbon conductors, so that these combinations are shorter than the ribbons of the resistor and located one opposite the other (Figure 5). These combinations are connected to an electric circuit via leads attached to the outside ribbon conductors (A, B, C and D). There can be one or two capacitive enclosures along the circumference of this complex system. They have a total of seven ribbons, dielectric and conductive. The capacitance can be adjusted by changing the length and width of the ribbons and the dielectric thickness.



**Slika 5**  
Möbiusov  
kondenzator  
**Figure 5**  
Möbius capacitor

A, B, C, D – vanjski oblozi /  
outer plates



Budući da je Möbiusovom kondenzatoru osnovica Möbiusov otpornik, za kojeg je spomenuto da ima kapacitivnu sastavnicu, razmotrit ćemo što se događa kad se na priključnice 3 otpornika (slika 3) priključi izmjenični napon  $u$  tjemene vrijednosti  $U_m$ . Neka je  $l$  duljina Möbiusova otpornika (kada se petlja prereže),  $c$  brzina širenja vala električnog polja između priključnica  $3_1$  i  $3_2$  otpornika i vrijeme  $T=l/c$  putovanja vala između  $3_1$  i  $3_2$ . Ako je frekvencija priključenog napona  $f$  bit će, u vremenu  $t$ , potencijal priključnice  $3_1$ :

$$U_{3_1} = U_m \sin 2\pi ft, \quad (4)$$

a priključnice  $3_2$ :

$$U_{3_2} = U_m \sin 2\pi f \left( t + \frac{l}{c} \right), \quad (5)$$

Napon na priključnicama 3 je jednak razlici potencijala priključnica  $3_1$  i  $3_2$ :

$$u = U_m \sin 2\pi ft - U_m \sin 2\pi f \left( t + \frac{l}{c} \right). \quad (6)$$

Otpornik će djelovati kao kondenzator kapaciteta  $C$ , pa je između priključnica 3 kapacitivna struja  $i_c=C(du/dt)$ . Derivacijom po vremenu  $t$  jednadžbe (6) i uvrštenjem  $f=c/l$ , dobiva se  $i_c=0$ . Stoga će priključivanjem Möbiusova otpornika u električni sklop, pri visokim frekvencijama, kada su valne duljine jednake ili blizu duljini petlje, ili njihovim cjelobrojnim višekratnicima, on propuštati rezonantnu frekvenciju i njezine harmonike, a prigušiti ostale frekvencije. Möbiusovi sustavi kondenzatora mogu, prema tvrdnji autora patenta, služiti za više svrha, među ostalim za filtriranje pravokutnih i pilastih valnih oblika, ispitivanje jednakosti i istofaznosti dvaju signala itd.

### 3.3 Möbiusov rezonator i filter

Pri vrlo visokim frekvencijama, kada su duljine vala reda veličine desetak centimetara ili manje, zbog velikih gubitaka, umjesto vodiča u obliku žica elektromagnetski valovi prostiru se valovodi-  
ma. To su metalne ili dielektrične, ali i mješovite,

Since the Möbius capacitor employs the principle of the Möbius resistor, which as previously mentioned has a capacitive component, we shall discuss what will happen when an alternating voltage  $u$  with a peak value  $U_m$  is applied between the connection points of the resistor according to Figure 3. Let  $l$  represent the length of the Möbius resistor (when the loop is cut), the propagation speed of the electric field wave between connection points  $3_1$  and  $3_2$  of the resistor, and  $T=l/c$  the wave propagation time between  $3_1$  and  $3_2$ . If  $f$  is the frequency of the applied voltage, then in time  $t$ , the potential of the connection point  $3_1$  is as follows:

and of the connection point  $3_2$ :

The voltage at 3 is equal to the potential difference between  $3_1$  and  $3_2$ :

The resistor will act as a capacitor of the capacitance  $C$ . Thus, current  $i_c=C(du/dt)$  flows between connection points  $3_1$  and  $3_2$ . The derivative of equation (6) with respect to time  $t$  and for  $f=c/l$ , is  $i_c=0$ . Therefore, when a Möbius resistor is placed in a high frequency electrical circuit, when the wavelengths are equal to or approximately the length of the loop or their integer multiples, it passes the resonant frequency and its harmonics and attenuates other frequencies. Möbius capacitors can, according to the inventor, serve many purposes, including the filtering of square and sawtooth waveforms, testing whether two signals are equal and in-phase etc.

### 3.3 Möbius resonator and filter

At very high frequencies, when the wavelength is of an order of magnitude of ten centimeters or less, due to high losses, instead of a conductor in the form of a wire, electromagnetic waves propagate in waveguides. These are metallic or dielectric, but also tubes of various cross-sectional dimensions [6]

cijevi različitih izmjera presjeka [6] i [7]. Presjeci valovoda, iz praktičnih su razloga pravokutni ili kružni. Izmjere valovoda ovise o frekvenciji. Čim je frekvencija niža, izmjere su valovoda veće. Energija se širi medijem unutar valovoda, refleksijama od zida do zida, a samo njezin manji dio ulazi u zidove i gubi se u obliku topline. Podjela valovoda temelji se na longitudinalnoj sastavnici polja usmjerenoj dužoj osi Z. Ako nema električnog polja u smjeru propagacije vala tada on nosi oznaku TE (transverzalni električni val), a onaj koji nema magnetskog polja u smjeru propagacije naziva se TM (transverzalni magnetski val). Jedan od poznatijih dielektričkih valovoda je svjetlovod, koji je danas u opsežnoj uporabi.

Valovod potpuno zatvoren sa svih strana, ispunjen dielektrikom sa zanemarivim gubicima i savršeno vodljivih zidova, ima svojstva elektromagnetskog rezonatora.

Jeffrey M. Pond patentirao je 3.9.2002. godine Möbiusov rezonator i filter [8]. Rezonator čini pravokutni valovod čiji je jedan kraj zakrenut uzduž osi za 180° i spojen s drugim krajem. To zakretanje dovodi do dodatnog faznog pomaka elektromagnetskog vala u valovodu što olakšava uvjete rezonancije u malom obujmu. Filtri mogu biti niskopropusni, visokopropusni i pojasnopropusni. Niskopropusni filtri propuštaju sve frekvencije od nulte do određene gornje granične frekvencije, a druge prigušuju. Visokopropusni filtri propuštaju sve frekvencije više od donje granične frekvencije, a pojasnopropusni propuštaju sve frekvencije između donje i gornje granične frekvencije. Filtri su građeni od kombinacija induktiviteta i kapaciteta. Mikrovalni filtri, osim onih načinjenih od prijenosnih linija, uključuju jedan ili više spojenih rezonatora s nizom dijafragmi (prozora) u valovodu koji djeluju kao induktivni ili kapacitivni elementi, kako bi se ostvarilo željeno frekvencijsko razdvajanje.

### 3.4 Namot elektromagneta

Nikola Tesla je 9.1.1894., kao svoj 56. patent u 18 godina, patentirao namot za elektromagnet [9]. U ovom patentu navodi da takvi svitci imaju zbog samoinduktiviteta značajnu jalovu sastavnicu, koja se može kompenzirati prikladnim kondenzatorima. Kako bi se izbjegla uporaba, u ono doba skupih i glomaznih kondenzatora, Tesla predlaže svoj svitak koji ne bi imao induktivnu sastavnicu, jer bi bila kompenzirana kapacitetom samoga svitka za određenu frekvenciju i napon. Inovacija se, prikazana načelnim primjerom, sastoji u tome da je zavojnica bifilarna, tj. paralelno se namataju dva izolirana vodiča A i B (slika 6). Kraj vodiča B spaja se na početak vodiča A. Razlika potencijala

and [7]. Waveguide cross sections are rectangular or circular for a practical reason. Waveguide dimensions depend on frequency. When the frequency is lower, waveguide dimensions are higher. Energy propagates through a medium in waveguides and is reflected from wall to wall. Only a small amount enters the walls and is lost in the form of heat. The classification of waveguides is based upon the longitudinal field component along axis Z. If there is no electric field in the direction of the propagation of the wave, it is designated as the transverse electric wave (TE), and if there is no magnetic field in the direction of the propagation of the wave, it is designated as the transverse magnetic wave (TM). One of the better known dielectric waveguides is an optical waveguide, which is widely used today.

A waveguide that is completely closed on all sides, filled with dielectric, with negligible losses and ideally conducting walls, has the properties of an electromagnetic resonator.

On September 3, 2002, Jeffrey M. Pond patented the Möbius resonator and filter [8]. The resonator consists of a rectangular waveguide, one end of which is twisted along its axis and connected to the other end. This twist provides additional phase shift of the electromagnetic wave in the waveguide, which facilitates a resonant condition in a smaller volume. Filters can be low-pass, band-pass or high-pass. Low-pass filters allow all frequencies from zero up to a cutoff frequency to pass through, and attenuate others. High-pass filters let through all frequencies higher than a low cutoff frequency, and band-pass filters let through all frequencies between the lower and upper cutoff frequencies. The filters consist of a combination of inductive and capacitive components. Microwave filters, with the exception of those consisting of transmission lines, include one or more connected resonators with low diaphragms (windows) in the waveguide that serve as inductive or capacitive elements, in order to achieve the desired frequency separation.

### 3.4 Coil for electromagnets

On January 9, 1894, Nikola Tesla obtained his 56<sup>th</sup> patent in 18 years, a coil for electromagnets [9]. In this patent, it is stated that such coils have a significant reactive component due to self-inductance, which can be compensated by suitable capacitors. In order to avoid the use of capacitors, which at the time were expensive and cumbersome, Tesla proposed a coil that would not have an inductive component, since it would be compensated for specific frequency and voltage by the capacity of the coil. The innovation, presented with a general example, consists of the fact that the coil is bifilar, i.e. with two insulated conductors A and B wound in parallel (Figure 6). The

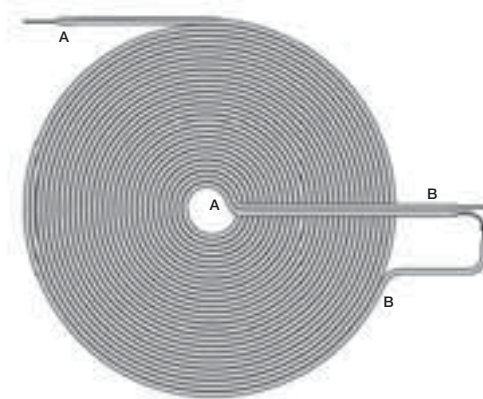
između bilo kojih susjednih točaka tih dvaju vodiča u zavojnici jednaka je polovici priključenog napon na svitke. Kod obične zavojnice razlika potencijala između dviju susjednih točaka dvaju zavoja jednaka je naponu priključenom na zavojnicu podijeljenom s ukupnim brojem zavoja. Zbog tijesno namotanih vodiča odijeljenih relativno tankom izolacijom, kapaciteti su veliki. Energija pohranjena u takvom kondenzatoru razmjerna je kvadratu razlike potencijala između obloga.

end of conductor B is connected to the starting point of conductor A. The potential difference between any neighboring points of these two conductors in the coil is equal to half the applied voltage to the coil. With ordinary coils, the potential difference between two contiguous points is equal to the applied voltage to the coil divided by the total number of turns (convolutions). Due to the tightly wound conductors, separated by relatively thin insulation, the capacities are high. The energy stored in such a capacitor is proportional to the square of the potential difference between adjacent turns.

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**Slika 6**

Načelo izrade namota  
elektromagneta  
Figure 6  
Coil for  
electromagnets



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Kako je razlika potencijala višestruko veća nego u običnoj zavojnici pa je i kompenzacija samoinduktiviteta razmjerno veća. Kompenzacija na ovaj način je pogodnija, jer su kapaciteti ravnomjerno raspoređeni. Ovisno o namjeni, svitci se mogu različito namatati i pritom postići željenu kompenzaciju.

Since the difference in potential is many times greater than in an ordinary coil, the compensation of the self-inductance is proportionally greater. Compensation is, thereby, improved because the capacities are evenly distributed. Depending upon the intended purpose, the coils can be wound in various ways in order to obtain the desired compensation.

## 4 ZAKLJUČAK

Nizu postupaka u izvedbi pasivnih dijelova električnih sklopova sa smanjenim parazitskim sastavnicama, od kojih su neki znani s kraja 19. stoljeća, pridružili su se novi. Suvremene izvedbe otpornika za visoke frekvencije, kao što su one u tehnici tankog filma, sendvič itd., većinom su prikladne za otpore većih od  $10 \Omega$  i relativno malih snaga.

Opisani Möbiusovi otpornici, prema svojoj izvedbi, čini se da su prikladni za male otpore i veće snage pri visokim frekvencijama. Jedna je od prednosti ovih otpornika i kondenzatora što se mogu različito oblikovati. Gotove vrpce otpornika i sustava kondenzatora mogu se omotati oko tijela različitih oblika, ili čak ih oblikovati u kugle, a da pritom nema međusobnih utjecaja pojedinih njihovih dijelova, ili utjecaja okolnih predmeta.

## 4 CONCLUSION

Procedures for devising electric circuit components with reduced parasitic components, some of which have been known since the end of the 19<sup>th</sup> century, are being joined by new ones. The majority of modern high frequency resistors, such as thin-film, sandwich etc., are suitable for resistance of greater than  $10 \Omega$  and relatively low power.

The Möbius resistors described appear to be more suitable for low resistance and high power at high frequencies, due to their construction. One of the advantages of these resistors and capacitors is that they can be shaped in various ways. Finished resistor strips and capacitor systems can be wound around objects of a variety shapes or even formed into a sphere, without the components being affected by each other or coupled to surrounding objects.

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