

# NUMERIČKI PRORAČUN NISKOFREKVENCIJSKIH ELEKTRO- MAGNETSKIH PRIJELAZNIH POJAVA U ENERGETSKIM TRANSFORMATORIMA THE NUMERICAL CALCULATION OF LOW FREQUENCY ELECTRO- MAGNETIC TRANSIENT PHENOMENA IN POWER TRANSFORMERS

Doc. dr. sc. Amir Tokić, Univerzitet u Tuzli, Fakultet elektrotehnike,  
Franjevačka 2, 75000 Tuzla, Bosna i Hercegovina

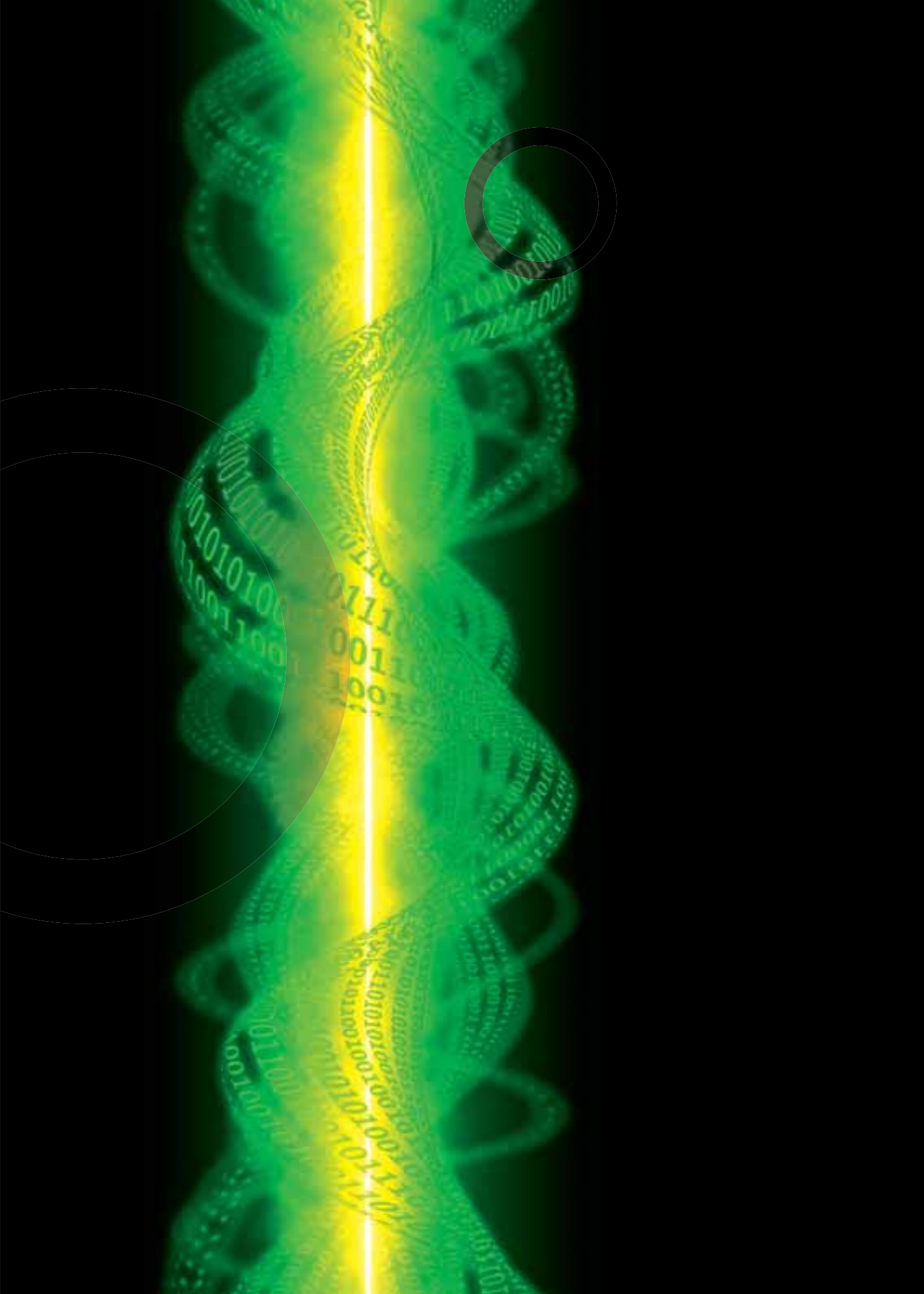
Prof. dr. sc. Ivo Uglešić, Sveučilište u Zagrebu,  
Fakultet Elektrotehnike i računarstva, Unska 3, 10000 Zagreb, Hrvatska

U radu je predstavljen model transformatora primjenjiv u niskofrekvencijskim elektromagnetskim prijelaznim pojavama, frekvencija reda oko 1 kHz. Analiziran je primjer uklapanja neopterećenog energetskog transformatora. Prvo je pokazan pojednostavljeni analitički pristup, a zatim, zbog njegova ograničenja, u analizu je uveden numerički pristup rješavanja krutih diferencijalnih jednadžbi koje opisuju prijelaznu pojavu. U oba slučaja je realiziran algoritam za generiranje valnih oblika varijabli stanja. Rezultati oba algoritma su uspoređeni s rezultatima MATLAB/Simulink/Power System Blockset, namjenskog programa za analizu elektromagnetskih prijelaznih pojava u elektroenergetskom sustavu. Razvijeni algoritam se može uspješno koristiti u ostalim niskofrekvencijskim prijelaznim pojavama gdje je glavni predmet analize nelinearni karakter transformatora: ferorezonancija, ispad tereta, kvarovi kod transformatora itd.

In this article, a transformer model is presented that is applicable to low frequency electromagnetic transient phenomena of up to 1 kHz. An example of the energization of a no-load transformer is analyzed. A simplified analytical approach is presented first. Due to the limitations of this approach, a numerical approach is introduced for the solution of the stiff differential equations that describe the transient phenomena. In both cases, algorithms have been developed for generating the waveforms of the state variables. The results of both algorithms are compared to the results of the MATLAB/Simulink/Power System Blockset for the analysis of electromagnetic transient phenomena in electrical energy systems. Developed algorithm with the introduced numerical approach can be used successfully in other low frequency transient phenomena where the main subject of analysis is the nonlinear character of the transformer: ferroresonance, load switch-off, transformer faults etc.

**Ključne riječi:** implicitno trapezno pravilo, krivulja magnetiziranja, krute diferencijalne jednadžbe, uklapanje transformatora

**Key words:** implicit trapezoidal rule, magnetization curve, stiff differential equations, transformer energization



## 1 UVOD

U niskofrekvencijske prijelazne pojave u transformatorima se obično ubrajaju uklapanje neopterećenih transformatora i ferorezonancija. U spomenutim prijelaznim pojavama parametar, koji dominantno utječe na rezultate prijelaznih pojava, je nelinearni induktivitet željezne jezgre transformatora. Kao posljedica nelinearnosti jezgre može doći do jakih strujnih udara prilikom uklapanja neopterećenih transformatora. Osnovne karakteristike struja uklopa transformatora su relativno velika amplituda, koja dostiže ekstremno i do  $10 I_{\text{naz}}$  kao i relativno velika duljina trajanja do postizanja stacionarnog stanja [1]. Ovakve struje često mogu uzrokovati nepotrebno djelovanje zaštitnih uređaja budući da mogu dostići vrijednosti struja kratkog spoja transformatora. S tim u vezi, danas su razvijene različite metode za razlikovanje struje uklapanja od struje kratkog spoja transformatora. Najčešće upotrebljavane tehnike razlikovanja su: harmonijska analiza struje (praćenje drugog harmonika struje) [2], energetske metode [3], metode magnetskih karakteristika transformatora [4], te suvremene tehnike koje se baziraju na upotrebi wavelet transformacije i neuronskih mreža [5], suvremenih korelacijskih algoritama [6], itd.

Posljedica struja uklapanja neopterećenih transformatora mogu biti privremeni, niskofrekvencijski, nesinusoidalni prenaponi [7], koji mogu značajno energetski preopteretiti metal-oksidne odvodnike prenapona koji su instalirani uz transformatore [8]. Zagrijavanje odvodnika bitno zavisi od promatrane konfiguracije mreže i od parametara sustava kao i odgovarajućih početnih uvjeta (trenutak uklopa transformatora, remanentni magnetizam transformatora itd.). Amplituda i duljina trajanja ovakvih privremenih napona, a samim time i zagrijavanje odvodnika prenapona, su znatno izraženiji u uvjetima slabih elektroenergetskih sustava.

Dodatno, uklapanje, odnosno isklapanje transformatora u električnim krugovima koje sadrže kapacitivnosti može dovesti do dugotrajnih ferorezonantnih prekostruja, odnosno prenapona. Tipični primjeri nastanka ferorezonancije nastupaju pri serijskoj kompenzaciji [9] ili pri isklapanju naponskih mjernih transformatora [10], te pri neregularnim sklopnim operacijama trofaznih prekidača (prijevremeni uklop/isklop jedne faze u mrežama sa izoliranom neutralnom točkom) [11].

S obzirom na iznesene praktične strane problema pri niskofrekvencijskim prijelaznim pojavama transformatora, potrebno je naročitu pozornost usmjeriti na pravilno modeliranje transformatora, odnosno simuliranje spomenutih pojava.

## 1 INTRODUCTION

Low frequency transient phenomena in transformers usually include the inrush currents of no-load transformers and feroresonance. In these transients, the parameter with the dominant impact on the results is the nonlinear inductance of the iron core. As a consequence of the nonlinearity of the core, high inrush currents can occur when energizing no-load transformers. The basic characteristics of transformer inrush currents are relatively high amplitude, which in extreme cases can reach up to  $10 I_{\text{rated}}$  as well as the relatively long period of time until a steady state is reached [1]. Such currents frequently can cause unnecessary tripping to occur because they can reach the short-circuit current values of the transformers. In connection with this, various methods have been developed today for differentiating between transformer inrush current and short-circuit current. The most frequently used differentiation techniques are as follows: harmonic analysis of the current (second harmonic component) [2], power differential methods [3], methods based on transformer magnetizing characteristics [4], modern techniques that are based on the use of wavelet transform and neural networks [5], modern correlation algorithms [6] etc.

Consequences of no-load transformer inrush currents can be temporary, low frequency and nonsinusoidal overvoltages [7], which can significantly overload the metal oxide surge arresters that are installed next to the transformers [8]. The thermal stress on surge arresters depends significantly on the network configuration and the system parameters as well as the initial conditions (the instant of energization, remnant magnetism etc.). The amplitude and duration of such transient voltages, together with the thermal stress on surge arresters, are significantly more marked under the conditions of weak powersystems.

Additionally, energizing and de-energizing transformers in electrical networks with capacitance can lead to long term overcurrents or overvoltages due to feroresonance. Typical examples of feroresonance occur in series compensation [9], when switching off voltage measuring transformers [10] and due to the abnormal switching operations of three-phase switches (the premature energizing/de-energizing of a transformer phase in networks with an isolated neutral point) [11].

Taking into account the above-described practical aspect of the problem with the low frequency transient phenomena of transformers, particular attention should be devoted to the correct modeling of transformers, i.e. the simulation of these phenomena.

Kao što je već spomenuto, osnovna poteškoća u modeliranju energetskih transformatora je nelinearni karakter induktiviteta željezne jezgre transformatora. Ostali parametri transformatora: otpor i rasipni induktiviteti primarnog i sekundarnog namota kao i otpor koji reprezentira gubitke u željezu uzimaju se konstantnim [12]. Osnovna krivulja magnetiziranja transformatora dana je na slici 1a. Ova krivulja se može kvalitativno aproksimirati s dva pravca, slika 1b, koji predstavljaju tangente u nezasićenom i zasićenom području. Krivulja magnetiziranja energetskih transformatora ima jako oštar prijelaz iz nezasićenog u zasićeno područje. Ovo je posljedica konstruktivne izvedbe visokonaponskih energetskih transformatora. Naime, s porastom naponske razine, odnosno nazivne snage transformatora struja praznog hoda se smanjuje i iznosi [13] oko 5 % do 10 % nazivne struje transformatora za transformatore snaga reda 100 kVA i opada sve do vrijednosti oko 0,47 % do 0,59 % nazivne struje transformatora za transformatore snaga reda 500 MVA.

Pregled standardnih vrijednosti struje magnetiziranja za energetske transformatore različitih nazivnih snaga dan je tablicom 1. Struja prijelaza u zasićeno područje  $i_z$  jednaka je nazivnoj struji pomnoženoj s faktorom ulaska u zasićenje  $k$ :  $i_z = k \cdot i_{0\text{naz}}$ , gdje je za energetske transformatore obično  $1,05 \leq k \leq 1,3$ . Dakle, koljeno krivulje magnetiziranja za energetske transformatore velikih snaga je razmješteno u veoma uskom području nazivne struje transformatora (reda 0,5 % do 1 %  $i_{\text{naz}}$ ). Površina, koja je omeđena realnom krivuljom magnetiziranja i njenom aproksimacijom preko dva pravca, ovdje je reda svega oko 0,001 % ako uzmemo da je 100 % površina ispod cijele krivulje magnetiziranja u p.u. sistemu. Logična je posljedica ovako malih nazivnih struja praznog hoda da predstavljanje krivulje magnetiziranja preko svega dva pravca čini gotovo zanemarive pogreške u usporedbi s realnim predstavljanjem krivulje [14].

As previously mentioned, the basic difficulty in modeling power transformers is the nonlinear character of the inductance of the iron transformer core. The other transformer parameters, the resistance and leakage inductance of the primary and secondary windings as well as the resistance that represents losses in iron are assumed to be constant [12]. The basic transformer magnetizing curve is presented in Figure 1a. This curve can be qualitatively approximated with two straight lines, Figure 1b, that represent tangents in the unsaturated and saturated regions. The power transformer magnetizing curve has a very sharp transition from the unsaturated to the saturated regions. This is a consequence of the design of high voltage power transformers. With an increase in the voltage level or rated power of the transformer, no-load current is decreased and amounts to approximately 5 % to 10 % of the transformer rated current [13] for transformers with power ratings of 100 kVA and decreases to a value of approximately 0,47 % to 0,59 % of transformer rated current for transformers with power ratings of 500 MVA.

A review of the standard values of power transformer magnetizing currents for various rated powers is presented in Table 1. The transition current to the saturated region  $i_z$  is equal to the rated current multiplied by the saturation factor  $k$ :  $i_z = k \cdot i_{\text{rated}}$ , where usually for power transformers  $1,05 \leq k \leq 1,3$ . Thus, the bend in the magnetizing curve for a high power transformer is in a very narrow region of the transformer rated current (an order of 0,5 % to 1 %  $i_{\text{rated}}$ ). The surface, which is bounded by the real magnetizing curve and its approximation by two straight lines is of an order here of only approximately 0,001 % if it is taken into account that 100 % of the surface below the magnetizing curve is in a p.u. system. A logical consequence of such low rated no-load currents is that it is possible to approximate the magnetizing curve using two straight lines with nearly negligible error in comparison to the real curve [14].

Tablica 1 – Tipične vrijednosti struje praznog hoda kao postotak nazivne struje za energetske transformatore  
Table 1 – Typical values of no-load current as a percentage of rated current for power transformers

$S_{\text{TR}}$ (MVA)	0,1	1,0	10	20	40	60
$i_0$ (% $i_{\text{naz/rated}}$ )	5,0 – 8,0	1,75 – 2,32	0,35 – 1,1	0,8 – 1,2	0,65 – 0,94	0,58 – 0,84
$S_{\text{TR}}$ (MVA)	80	100	150	200	300	500
$i_0$ (% $i_{\text{naz/rated}}$ )	0,54 – 0,77	0,51 – 0,73	0,47 – 0,67	0,51 – 0,64	0,49 – 0,61	0,47 – 0,59

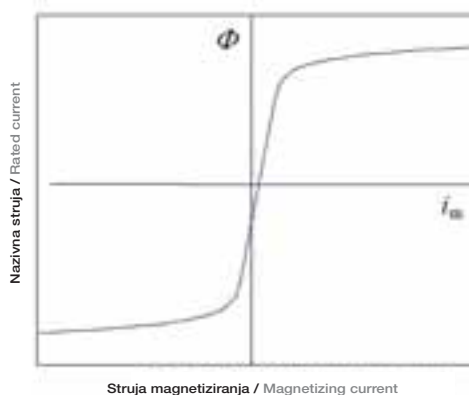
**Slika 1**

Krivulja magnetiziranja transformatora a) i njena aproksimacija preko dva pravca b)

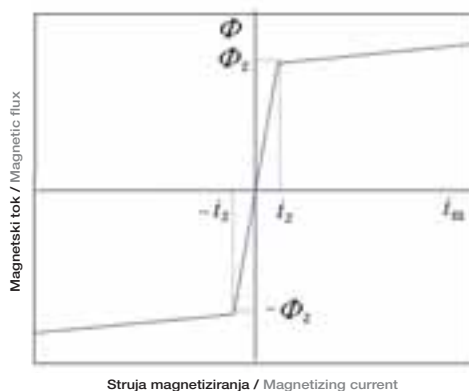
**Figure 1**

Magnetizing curve of transformer a) and its approximation via two straight lines b)

a)

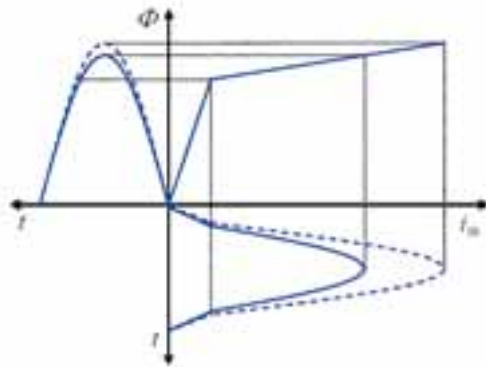


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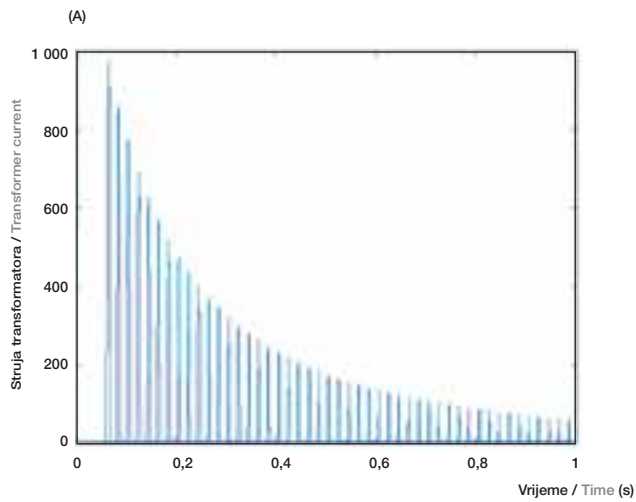


Posljedica nelinearnog karaktera željezne jezgre transformatora je nesinusoidalna struja magnetiziranja transformatora pri sinusoidalnom naponu napajanja, tj. sinusoidalnom magnetskom toku, što je jasno ilustrirano na slici 2. Na istoj slici su prikazana dva oblika struje magnetiziranja za dvije različite tjemene vrijednosti magnetskog toka. Tijekom prijelazne pojave uklapanja transformatora kada magnetski tok može poprimiti vrijednosti i veće od svoje dvostruke nazivne vrijednosti [15], dolazi do jakih strujnih udara transformatora, što je pokazano na slici 3.

A consequence of the nonlinear character of the iron transformer core is the nonsinusoidal transformer magnetizing current at the sinusoidal supply voltage, i.e. sinusoidal magnetic flux, which is clearly illustrated in Figure 2. In the same figure, two forms of magnetizing current are presented for two different peak values of the magnetic flux. During the transient phenomena of transformer energization, when the magnetic flux can acquire values greater than twice its rated value [15], high inrush current occurs, as presented in Figure 3.



**Slika 2**  
Nesinusoidalna  
struja magnetiziranja  
transformatora  
Figure 2  
Nonsinusoidal  
transformer  
magnetizing current



**Slika 3**  
Tipičan valni oblik  
struje uklapanja  
transformatora  
Figure 3  
Typical waveform  
of transformer  
inrush current

## 2 MATEMATIČKI MODEL PRI UKLAPANJU NEOPTEREĆENOG ENERGETSKOG TRANSFORMATORA

U ovom poglavlju će se analizirati matematički model pri uklapanju neopterećenog energetskog transformatora, slika 4a i b. Između transformatora i točke priključka na mrežu postoji kapacitet  $C$ , kojim se ekvivalentira prilaz kabelskim ili nadzemnim vodovima, kapacitet kondenzatorskih baterija i sl.

## 2 MATHEMATICAL MODEL OF THE ENERGIZATION OF A NO-LOAD POWER TRANSFORMER

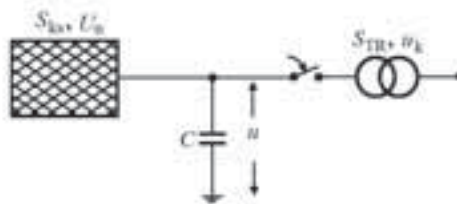
This chapter presents an analysis of a mathematical model of the energization of a no-load power transformer, Figures 4a and b. Between the transformer and the connection point to the network, there is a capacity  $C$ , which is equivalent to the capacity of a cable or overhead approach line, the capacitance of the capacitors etc.

**Slika 4**

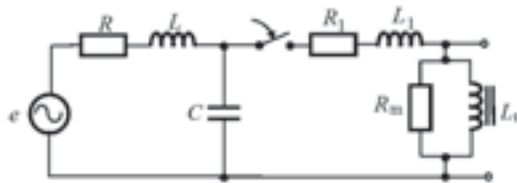
Uklapanje neopterećenog energetskog transformatora

Figure 4

The energization of a no-load power transformer



a) Shema uklapanja transformatora na mrežu / Simplified electrical circuit of transformer energization



b) Odgovarajuća zamjenska shema slike a) / Equivalent electrical circuit of Figure a)

Parametri mreže se dobivaju iz podataka o snazi topolnog kratkog spoja u točki priključka:

The network parameters are obtained from data on the three-phase short-circuit power at the point of connection:

$$L = \frac{U_n^2}{S_{ks} \cdot \omega} \quad (1)$$

Pretpostavlja se da je odnos  $X/R$  za danu mrežu poznat, na osnovi kojega se određuje otpor  $R$ .

It is assumed that the ratio of  $X/R$  for the given network is known, on the basis of which we determine resistance  $R$ .

Na osnovi poznate nazivne snage transformatora  $S_{TR}$ , te napona kratkog spoja moguće je odrediti rasipni induktivitet transformatora:

On the basis of the known transformer rated power  $S_{TR}$  and the short-circuit voltage, it is possible to determine the leakage inductance of the transformer:

$$L_1 = \frac{U_n^2}{S_{TR} \cdot \omega} \cdot u_k \quad (2)$$

Ostali podaci, djelatni otpor primarnog namota transformatora  $R_1$ , djelatni otpor izazvan gubicima u željezu  $R_m$ , te nelinearni induktivitet željezne

Other data, the effective resistance of the primary transformer winding  $R_1$ , the effective resistance due to iron core losses  $R_m$ , and the nonlinear inductance

jezgre transformatora  $L_m$ , inače definirana krivuljom magnetiziranja  $\Phi-i_m$  ( $\Phi$  je glavni ulančeni magnetski tok,  $i_m$  je struja magnetiziranja transformatora), lako se mogu odrediti mjerenjem ili su već dani od strane proizvođača. Ovaj nelinearni induktivitet aproksimiran je s dva pravca u  $\Phi-i_m$  koordinatnom sustavu, slika 2, što je za energetske transformatore u praksi uglavnom prihvatljivo. Točka  $(i_z, \Phi_z)$  predstavlja kritičnu točku pri prelasku iz nezasićenog u zasićeno područje željezne jezgre. Koeficijenti pravaca ustvari predstavljaju induktivite u nezasićenom ( $L_{m1}, \text{const}_1$ ) i zasićenom području ( $L_{m2}, \text{const}_2$ ). Na ovaj način dobivamo funkcionalnu ovisnost struje magnetiziranja o magnetskom toku kao (funkcijom *sign* osiguravamo pozicioniranje u odgovarajućem kvadrantu):

$$i_m = \frac{1}{L_{m1}} \Phi, |\Phi| \leq \Phi_z, \quad (3)$$

$$i_m = \frac{1}{L_{m2}} \Phi + \text{sign}(\Phi) \left( \frac{1}{L_{m1}} - \frac{1}{L_{m2}} \right) \Phi_z, |\Phi| > \Phi_z, \quad (4)$$

of the transformer iron core  $L_m$ , otherwise defined by the magnetizing curve  $\Phi-i_m$  ( $\Phi$  is the main linkage magnetic flux,  $i_m$  is the transformer magnetizing current), can easily be determined through measurement or are already provided by the manufacturer. This nonlinear inductance is approximated with two straight lines in the  $\Phi-i_m$  coordinate system, Figure 2, which is generally acceptable in practice for power transformers. Point  $(i_z, \Phi_z)$  represents the critical point at the transition from the unsaturated region to the saturated region of the iron core. The straight lines coefficients actually represent inductances in the unsaturated region  $L_{m1}, \text{const}_1$  and saturated region  $L_{m2}, \text{const}_2$ . In this manner, we obtain the functional dependence of the magnetizing current on the magnetic flux (we define the position in the corresponding quadrant with the *sign* function) as follows:

### 3 ANALITIČKI PRISTUP

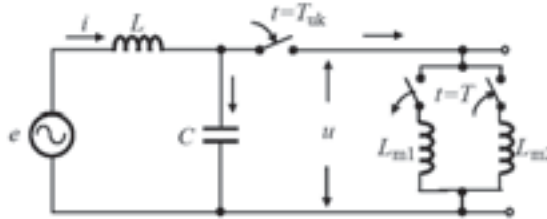
Radi jednostavnosti zanemarit će se sve aktivne elemente sa slike 4b. Dakle, pri prijelaznoj pojavi uklapanja transformatora ako trenutna vrijednost magnetskog toka, po apsolutnoj vrijednosti premaši kritičnu vrijednost  $\Phi_z$ , induktivnost transformatora se mijenja sa  $L_{m1}$  na  $L_{m2}$ . To ustvari znači da se trenutak kada magnetski tok dostigne vrijednost  $\Phi_z$  može uzeti kao vrijeme  $t=T>T_{\text{uk}}$  isklapanja induktiviteta  $L_{m1}$  odnosno uklapanja induktiviteta  $L_{m2}$ . Ekvivalentna shema bi u tom slučaju izgledala kao na slici 5. Slično, pri smanjenju magnetskog toka ispod vrijednosti  $\Phi_z$  isključuje se induktivitet  $L_{m2}$  i uključuje induktivitet  $L_{m1}$ .

### 3 ANALYTICAL APPROACH

For the purpose of simplicity we shall ignore all the active elements from Figure 4b. In the transient phenomena during transformer energization, if the instantaneous value of the magnetic flux in terms of absolute value exceeds the critical value of  $\Phi_z$ , the transformer inductance changes from  $L_{m1}$  to  $L_{m2}$ . This actually means that the moment when the magnetic flux reaches the value of  $\Phi_z$  can be taken as the time  $t=T>T_{\text{uk}}$  of switching off the inductance  $L_{m1}$ , respectively the time of switching on the inductance  $L_{m2}$ . In this case, an equivalent diagram would look like the one presented in Figure 5. Similarly, when the magnetic flux is decreased below the value of  $\Phi_z$ , inductance  $L_{m2}$  is switched off and inductance  $L_{m1}$  is switched on.



**Slika 5**  
Uklapanje transformatora – pojednostavljeni model  
Figure 5  
Transformer energization – simplified model



Dakle, promjena trenutačne vrijednosti magnet-skog toka pri prelaznoj pojavi uvjetuje prelasku s jednog na drugi pravac što u diferencijalnim jednađbama koje opisuju ponašanje električnih krugova ustvari znači promjenu koeficijenata. Proces počinje s induktivitetom  $L_{m1}$ . Uz oznake kao na slici 5 dobiju se diferencijalne jednađbe koje opisuju ponašanje električnog kruga u proizvoljnom trenutku  $t \geq T_{uk}$ , gdje je  $t = T_{uk}$  trenutak uklapanja prekidača :

Therefore, in transient phenomena transition from one straight line to another is conditional upon change in the instantaneous value of the magnetic flux, which in differential equations that describe the behavior of electric circuits actually signifies coefficient changes. The process begins with the inductance  $L_{m1}$ . Using the same symbols as in Figure 5, differential equations are obtained that describe the behavior of the electric circuit at the arbitrary moment  $t \geq T_{uk}$ , where  $t = T_{uk}$  is the moment that the switch is turned on:

$$E_m \cos \omega t = \frac{d^3 \Phi}{dt^3} + \frac{1}{C} \left( \frac{1}{L_{m1}} + \frac{1}{L} \right) \frac{d\Phi}{dt}, \quad |\Phi| \leq \Phi_s, \quad (5)$$

$$E_m \cos \omega t = \frac{d^3 \Phi}{dt^3} + \frac{1}{C} \left( \frac{1}{L_{m2}} + \frac{1}{L} \right) \frac{d\Phi}{dt}, \quad |\Phi| > \Phi_s. \quad (6)$$

Rješava se prvo diferencijalna jednađba (5). Uz izraz za prirodnu kružnu frekvenciju kruga:

Differential equation (5) is solved first. When the natural circular frequency of the circuit is as follows:

$$\omega_{01} = \sqrt{\frac{1}{C} \left( \frac{1}{L_{m1}} + \frac{1}{L} \right)}, \quad (7)$$

dobiva se opće rješenje diferencijalne jednađbe (5):

the general solution to equation (5) is obtained:

$$\Phi(t) = a + b \cos \omega_{01} t + c \sin \omega_{01} t + B \sin \omega t. \quad (8)$$

Konstante  $a, b$  i  $c$  određuju se iz početnih uvjeta:

Constants  $a, b$  and  $c$  are determined from the initial conditions:

$$\Phi(T_{uk}) = \Phi_0, \quad (9)$$

$$u(T_{uk}) = \left. \frac{d\Phi}{dt} \right|_{t=T_{uk}} = U_0, \quad (10)$$

$$i_C(T_{uk}) = C \left. \frac{du}{dt} \right|_{t=T_{uk}} = C \left. \frac{d^2\Phi}{dt^2} \right|_{t=T_{uk}} = I_{C0}. \quad (11)$$

Napon  $U_0$  i struja  $I_{C0}$  se određuju iz stanja prije uklanjanja prekidača:

Voltage  $U_0$  and current  $I_{C0}$  are determined from the state prior to turning on the switch:

$$U_0 = \frac{E_m}{1 - \omega^2 LC} \cos \omega T_{uk}, \quad (12)$$

$$I_{C0} = -\frac{\omega CE_m}{1 - \omega^2 LC} \sin \omega T_{uk}. \quad (13)$$

Koeficijenti  $a$ ,  $b$  i  $c$  se dobivaju iz matrične jednadžbe koja se formira na osnovi jednadžbi (9) do (13):

Coefficients  $a$ ,  $b$  and  $c$  are obtained from the matrix equation formed on the basis of equations (9) to (13):

$$M = K^{-1} \cdot N, \quad (14)$$

gdje su matrice  $M$ ,  $K$  i  $N$  redom:

where matrices  $M$ ,  $K$  and  $N$  are as follows:

$$M = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad (14a)$$

$$K = \begin{bmatrix} 1 & \cos \omega_{01} T_{uk} & \sin \omega_{01} T_{uk} \\ 0 & -\omega_{01} \sin \omega_{01} T_{uk} & \omega_{01} \cos \omega_{01} T_{uk} \\ 0 & -\omega_{01}^2 \cos \omega_{01} T_{uk} & -\omega_{01}^2 \sin \omega_{01} T_{uk} \end{bmatrix}, \quad (14b)$$

$$N = \begin{bmatrix} \Phi_0 - B \sin \omega T_{uk} \\ U_0 - B\omega \cos \omega T_{uk} \\ I_{C0} / C + B\omega^2 \sin \omega T_{uk} \end{bmatrix}. \quad (14c)$$

Kada se odrede konstante  $a$ ,  $b$  i  $c$  tada se za vremenski oblik magnetskog toka  $\Phi(t)$ , napona  $u(t)$  na transformatoru i struje kondenzatora  $i_c(t)$  dobiva:

$$\Phi(t) = a + b \cos \omega_{01} t + c \sin \omega_{01} t + B \sin \omega t, \quad (15)$$

$$u(t) = -b \omega_{01} \sin \omega_{01} t + c \omega_{01} \cos \omega_{01} t + B \omega \cos \omega t, \quad (16)$$

$$i_c(t) = C(-b \omega_{01}^2 \cos \omega_{01} t - c \omega_{01}^2 \sin \omega_{01} t - B \omega^2 \sin \omega t), \quad (17)$$

U jednadžbi (8) konstanta  $a$  predstavlja istosmjernu komponentu magnetskog toka,  $b$ ,  $c$  i  $\omega_{01}$  konstante kojima je definiran vlastiti odziv, a  $B$  i  $\omega$  konstante kojima je definiran prinudni odziv u rješenju za magnetski tok. Posljednje tri jednadžbe vrijede sve dok je zadovoljeno  $|\Phi| \leq \Phi_z$ . U protivnom, kada bude  $|\Phi| > \Phi_z$  tada ponašanje električnog kruga opisuje diferencijalna jednadžba (6) opisuje stanje ravnoteže s parametrom  $L_{m2}$  umjesto  $L_{m1}$ . Početni uvjeti za novu diferencijalnu jednadžbu (6) su posljednje trenutačne vrijednosti rješenja prvobitne jednadžbe (5). Rješenja diferencijalne jednadžbe (6) se dobivaju istim postupkom kao rješenja jednadžbe (5). Analogno se razmišlja pri ponovnom smanjenju magnetskog toka ispod vrijednosti  $\Phi_z$  sa odgovarajućim početnim uvjetima koji su određeni posljednjim trenutačnim vrijednostima stare diferencijalne jednadžbe.

Struja magnetiziranja transformatora  $i_m$  određuje se na osnovi relacija (3) i (4).

Generalno se može organizirati algoritam koji bi prateći vrijednost trenutačnog magnetskog toka rješavao diferencijalne jednadžbe (5) i (6) s odgovarajućim početnim uvjetima. Uvodeći težinske koeficijente  $k_1$  i  $k_2$  koji bi uzimali vrijednosti 0 ili 1 moguće je načiniti petlju koja bi stalno računala vrijednost magnetskog toka u zasićenom ili nezasićenom području vodeći računa o odgovarajućim početnim uvjetima.

Remanentni magnetizam transformatora  $\Phi_{rem}$  moguće je uvažiti ako se u proračun krene s tom vrijednošću,  $\Phi(T_{uk}) = \Phi_{rem}$ . Histerezna petlja je zanemarena, što je u praktičkim primjerima za energetske transformatore sasvim prihvatljivo [16].

When constants  $a$ ,  $b$  and  $c$  are determined, the magnetic flux  $\Phi(t)$ , transformer voltage  $u(t)$  and condenser current  $i_c(t)$  waveforms as a function of time are then as follows:

In equation (8), constant  $a$  represents the DC component of the magnetic flux,  $b$ ,  $c$  and  $\omega_{01}$  are the constants by which the self response is defined, and  $B$  and  $\omega$  are the constants by which the forced response is defined in the solution for the magnetic flux. The last three equations are valid until the condition of  $|\Phi| \leq \Phi_z$  is met. Otherwise, when  $|\Phi| > \Phi_z$ , differential equation (6) describes the behavior of the electric circuit with the parameter  $L_{m2}$  instead of  $L_{m1}$ . The initial conditions for a new differential equation (6) are the last instantaneous values from the solution of equation (5). The solution to differential equation (6) is obtained according to the same procedure as the solution to equation (5). An analogous procedure is used when the magnetic flux is again reduced below the value of  $\Phi_z$  with the corresponding initial conditions that are determined using the instantaneous values of the previous differential equation.

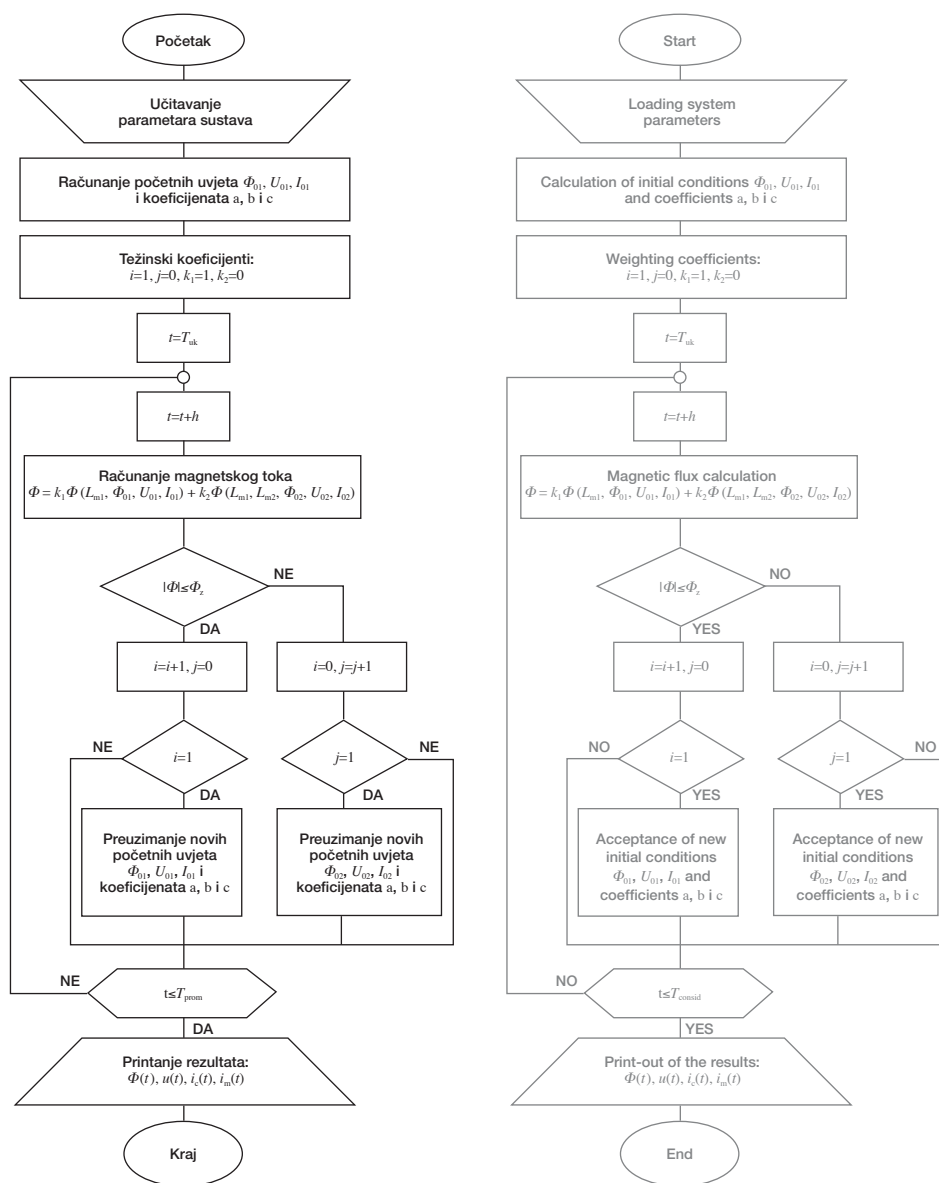
The transformer magnetizing current  $i_m$  is determined on the basis of equations (3) and (4).

It is generally possible to develop an algorithm that would solve differential equations (5) and (6) by following the value of the instantaneous magnetic flux with the corresponding initial conditions. By introducing weighting coefficients  $k_1$  and  $k_2$  that would assume the value of 0 or 1, it is possible to make a loop that that would constantly calculate the value of the magnetic flux in the saturated and unsaturated regions, taking the corresponding initial conditions into account.

Transformer remnant magnetism,  $\Phi_{rem}$  can be taken into account if this value is entered into the calculation  $\Phi(T_{uk}) = \Phi_{rem}$ . The hysteresis loop is ignored, which is completely acceptable in practical applications for power transformers [16].

Pojednostavljeni algoritam računanja varijabli stanja (magnetskog toka, napona i struje) dan je na slici 6. Otežavajuća činjenica pri realizaciji programa je da se početni uvjeti pri svakom prijelazu iz jednog u drugo područje stalno moraju preračunavati.

A simplified algorithm for the calculation of state variables (magnetic flux, voltage and current) is presented in Figure 6. A complicating factor in the implementation of the program is that the initial conditions at every transition from one region to another must constantly be recalculated.



**Slika 6**  
Razvijeni algoritam, pojednostavljeni model  
Figure 6  
Developed algorithm, simplified model

## 4 NUMERIČKI PRISTUP

Sada će se analizirati slučaj uklapanja transformatora sa svim elementima prema slici 7. Dok se transformator još uvijek nalazi u nezasićenom području, gdje će se radi jednostavnosti induktivitet željezne jezgre označiti s  $L_m$ , vrijede jednačbe:

$$e = E_m \cos \omega t = R \cdot i + L \frac{di}{dt} + u_C, \quad (18)$$

$$i = i_C + i_1 = C \frac{du_C}{dt} + i_1, \quad (19)$$

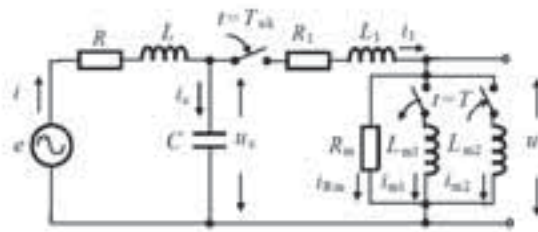
$$i_1 = i_{\text{magn}} + i_m = \frac{1}{R_m} u + \frac{1}{L_m} \Phi = \frac{1}{R_m} \frac{d\Phi}{dt} + \frac{1}{L_m} \Phi, \quad (20)$$

$$u_C = R_1 \cdot i_1 + L_1 \frac{di_1}{dt} + u = R_1 \cdot i_1 + L_1 \frac{di_1}{dt} + \frac{d\Phi}{dt}. \quad (21)$$

## 4 NUMERICAL APPROACH

A case will now be analyzed of the energization of a transformer with all the elements according to Figure 7. While the transformer is still in the unsaturated region, where for purposes of simplicity the inductance of the iron core will be designated by  $L_m$ , the following equations apply:

**Slika 7**  
Uklapanje transformatora  
– potpuni model  
Figure 7  
Transformer energization  
– complete model



Transformacijom posljednje četiri jednačbe dolazimo do diferencijalne jednačbe četvrtog reda oblika:

Through the transformation of the last four equations, we arrive at fourth-order differential equations:

$$e = a_4 \frac{d^4 \Phi}{dt^4} + a_3 \frac{d^3 \Phi}{dt^3} + a_2 \frac{d^2 \Phi}{dt^2} + a_1 \frac{d\Phi}{dt} + a_0 \Phi, \quad (22)$$

gdje su konstante  $a_i$ ,  $i = 0, 1, 2, 3, 4$  dane sa:

where the constants  $a_i$ ,  $i = 0, 1, 2, 3, 4$  are as follows:

$$a_0 = \frac{R}{L_m} + \frac{R_1}{L_m}, \quad (22a)$$

$$a_1 = R \left( \frac{C \cdot R_1}{L_m} + \frac{1}{R_m} \right) + \frac{L}{L_m} + \frac{R_1}{R_m} + \frac{L_1}{L_m} + 1, \quad (22b)$$

$$a_2 = R \cdot C \left( \frac{R_1}{R_m} + \frac{L_1}{L_m} + 1 \right) + L \left( \frac{C \cdot R_1}{L_m} + \frac{1}{R_m} \right) + \frac{L_1}{R_m}, \quad (22c)$$

$$a_3 = \frac{R \cdot C \cdot L_1}{R_m} + L \cdot C \left( \frac{R_1}{R_m} + \frac{L_1}{L_m} + 1 \right), \quad (22d)$$

$$a_4 = \frac{L \cdot C \cdot L_1}{R_m}. \quad (22e)$$

Uz realne podatke [8]:

with real data [8]:

- $E_m = 172$  kV,
- $R = 8,82$   $\Omega$ ,
- $L = 0,281$  H,
- $C = 4,218$   $\mu$ F,
- $R_1 = 0,529$   $\Omega$ ,
- $L_1 = 0,126$  H,
- $\Phi_z = 657,88$  W,
- $L_{m1} = 185,24$  H,
- $L_{m2} = 0,253$  H,
- $R_m = 0,576 \cdot 10^6$   $\Omega$ ,

- $E_m = 172$  kV,
- $R = 8,82$   $\Omega$ ,
- $L = 0,281$  H,
- $C = 4,218$   $\mu$ F,
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- $L_1 = 0,126$  H,
- $\Phi_z = 657,88$  W,
- $L_{m1} = 185,24$  H,
- $L_{m2} = 0,253$  H,
- $R_m = 0,576 \cdot 10^6$   $\Omega$ ,

primjenjujući Bairstow numeričku metodu, za korjene karakteristične jednadžbe bi dobili rješenja:

and by applying the Bairstow numerical method, the roots of the characteristic equation are obtained as follows:

$$p_1 = -4,575 \cdot 10^6 = -\alpha, \quad (23)$$

$$p_2 = -0,0504 = -\beta, \quad (24)$$

$$p_{3,4} = -15,876 \pm j919,096 = -\gamma \pm j\omega_0. \quad (25)$$

Tada je opće rješenje jednadžbe (22):

Then the general solution of the equation (22) is:

$$\Phi(t) = a \cdot e^{-\alpha t} + b \cdot e^{-\beta t} + e^{-\gamma t} (c \cdot \cos \omega_0 t + d \cdot \sin \omega_0 t) + B \cdot \sin(\omega t + \Psi). \quad (26)$$

Očita je jaka rasutost korijena  $p_1$  i  $p_2$ . Kada bi po analogiji na razmatranje kao u poglavlju 3 formirali matrice  $M$ ,  $K$  i  $N$ , lako bismo zaključili da matrica  $K$  predstavlja singularnu matricu, jer u prvom stupcu sadrži umnoške broja  $e^{p_i T_{uk}}$ . Svi ti brojevi su za računalo praktički jednaki nuli zbog goleme vrijednosti  $p_1$ . Dakle, klasičan analitički pristup u općem se slučaju ne bi mogao provesti zbog singularnosti matrice  $K$ . U daljem dijelu razmatrat će se numerički pristup rješavanja ovog problema. Međutim, i pri numeričkom pristupu veoma je važno obratiti pozornost na izbor odgovarajućeg numeričkog postupka zbog istaknute činjenice da su korijeni karakteristične jednadžbe jako rasuti u lijevom dijelu kompleksne ravnine. Jaka disperzija korijena karakteristične jednadžbe definira zasebnu klasu diferencijalnih jednadžbi poznatih pod imenom krute diferencijalne jednadžbe (*stiff differential equations*). Naime, diskutabilna je apsolutna stabilnost numeričkih postupaka primijenjenih na ovu vrstu jednadžbi [17]. Nijedan od klasičnih numeričkih postupaka u eksplicitnoj formi, bio jednokoračni ili višekoračni tipa Eulera, Runge-Kuta, Adams-Moulton itd. pri standardnim koracima integracije, ne osigurava apsolutnu stabilnost postupka, što dovodi do divergiranja rješenja u numeričkom smislu (pogreška  $\delta_k$  u  $k$ -toj iteraciji izaziva u  $k+1$ -oj iteraciji pogrešku  $\delta_{k+1} > \delta_k$ ). Apsolutnu stabilnost osiguravaju jedino implicitni numerički postupci [17] i [18], mada i pri njihovoj upotrebi treba biti jako oprezan. U konkretnom primjeru upotrijebljeno je apsolutno stabilno implicitno trapezno pravilo.

High dispersion of the roots  $p_1$  and  $p_2$  is evident. If matrices  $M$ ,  $K$  and  $N$  are formed, analogically to the discussion in Chapter 3, it could be easily concluded that matrix  $K$  represents a singular matrix because it contains multiples of the number  $e^{p_1 T_{uk}}$  in the first column. All these numbers are practically equal to zero for a computer due to the enormous value of  $p_1$ . Therefore, the classical analytical approach in a general case could not be implemented due to the singularity of matrix  $K$ . The numerical approach to the solution of this problem will be discussed subsequently. However, with the numerical approach it is very important to pay attention to the selection of the suitable numerical approach procedure due to the significant fact that the roots of the characteristic equation are highly dispersed in the left part of the complex plain. The high root dispersion of the characteristic equation defines a separate class of differential equations known as stiff differential equations. The absolute stability of the numerical approaches applied in this type of equation is disputable [17]. None of the classical numerical approaches in explicit form, whether of the single-step or multistep Euler, Runge-Kutta, Adams-Moulton type etc. using standard integration steps, assures the absolute stability of the procedure, which leads to divergence of the solutions in the numerical sense (error  $\delta_k$  in the  $k$ -th iteration causes error  $\delta_{k+1} > \delta_k$  in the  $k+1$  iteration). Only implicit numerical procedures guarantee absolute stability [17] and [18], although it is necessary to use them very cautiously. In a concrete example, the absolutely stable implicit trapezoidal rule is applied.

Jednadžbe (18) do (22) će se napisati u prostoru stanja uzimajući da je vektor varijabli stanja:

Equations (18) to (22) can be written in the state space form where the state variable vector is:

$$\mathbf{x} = [i \quad u_c \quad i_1 \quad \Phi]^T. \quad (27)$$

Za nezasićeno područje gdje inače vrijedi:

For the unsaturated region where the following otherwise applies:

$$i_m = \frac{1}{L_{m1}} \Phi \quad \text{za / for} \quad |\Phi| \leq \Phi_s, \quad (28)$$

dobiva se jednačba u prostoru stanja:

the following state space equation is obtained:

$$\dot{x} = A_1 x + b_1, \quad (29)$$

gdje su:

Where:

$$A_1 = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_1} & -\left(\frac{R_1}{L_1} + \frac{R_m}{L_1}\right) & \frac{R_m}{L_1 L_{m1}} \\ 0 & 0 & R_m & -\frac{R_m}{L_{m1}} \end{bmatrix}, \quad (29a)$$

$$b_1 = \left[ \frac{1}{L} E_m \cos \omega t \ 0 \ 0 \ 0 \right]^T. \quad (29b)$$

U zasićenom području gdje inače vrijedi relacija:

In the saturated region where the following relation otherwise applies:

$$i_m = \frac{1}{L_{m2}} \Phi + \text{sign}(\Phi) \left( \frac{1}{L_{m1}} - \frac{1}{L_{m2}} \right) \Phi_s \quad \text{za / for} \quad |\Phi| > \Phi_s, \quad (30)$$

dobiva se:

the following is obtained:

$$\dot{x} = A_2 x + b_2, \quad (31)$$



gdje su:

where:

$$A_2 = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_1} & -\left(\frac{R_1}{L_1} + \frac{R_m}{L_1}\right) & \frac{R_m}{L_1 L_{m2}} \\ 0 & 0 & R_m & -\frac{R_m}{L_{m2}} \end{bmatrix}. \quad (31a)$$

$$b_2 = \left[ \frac{1}{L} E_m \cos \omega t \ 0 \ \frac{R_m}{L_1} \text{sign}(\Phi) \left( \frac{1}{L_{m1}} - \frac{1}{L_{m2}} \right) \Phi_1 - R_m \text{sign}(\Phi) \left( \frac{1}{L_{m1}} - \frac{1}{L_{m2}} \right) \Phi_2 \right]^T. \quad (31b)$$

Implicitno trapezno pravilo primijenjeno na sustav  $\dot{x} = A_i x + b_i$ ,  $i = 1, 2$  daje iteracijsku vezu:

The implicit trapezoidal rule applied to the system  $\dot{x} = A_i x + b_i$ ,  $i = 1, 2$  yields the iteration expression:

$$x_{i+1} = \left[ E - \frac{h}{2} A_i \right]^{-1} \left( \left[ E + \frac{h}{2} A_i \right] x_i + \frac{h}{2} [b_i(t_i) + b_i(t_{i+1})] \right) \quad (32)$$

U posljednjoj relaciji sa  $h$  označen je korak integracije i on je veoma problematičan za eksplicitne metode. Naime, da bi se osigurala stabilnost ovih postupaka korak se mora održati dovoljno malim da bi testovi stabilnosti bili zadovoljeni. Za Eulerovo pravilo je potrebno da korak integracije bude

In the previous expression,  $h$  is designated as the integration step and it is highly problematic for explicit methods. In order to assure the stability of these procedures, the step should be kept sufficiently small in order to satisfy the stability tests. For Euler's rule, it is necessary for the integration step to be:

$$h \leq \min_i \left\{ \frac{2 \text{Re}(\lambda_i)}{|\lambda_i|^2} \right\}, \quad (33)$$

gdje su sa  $\lambda_i$  označene sve svojstvene vrijednosti matrice  $A_i$ . Za ovaj bi primjer već za nezasićeno područje korak bio:

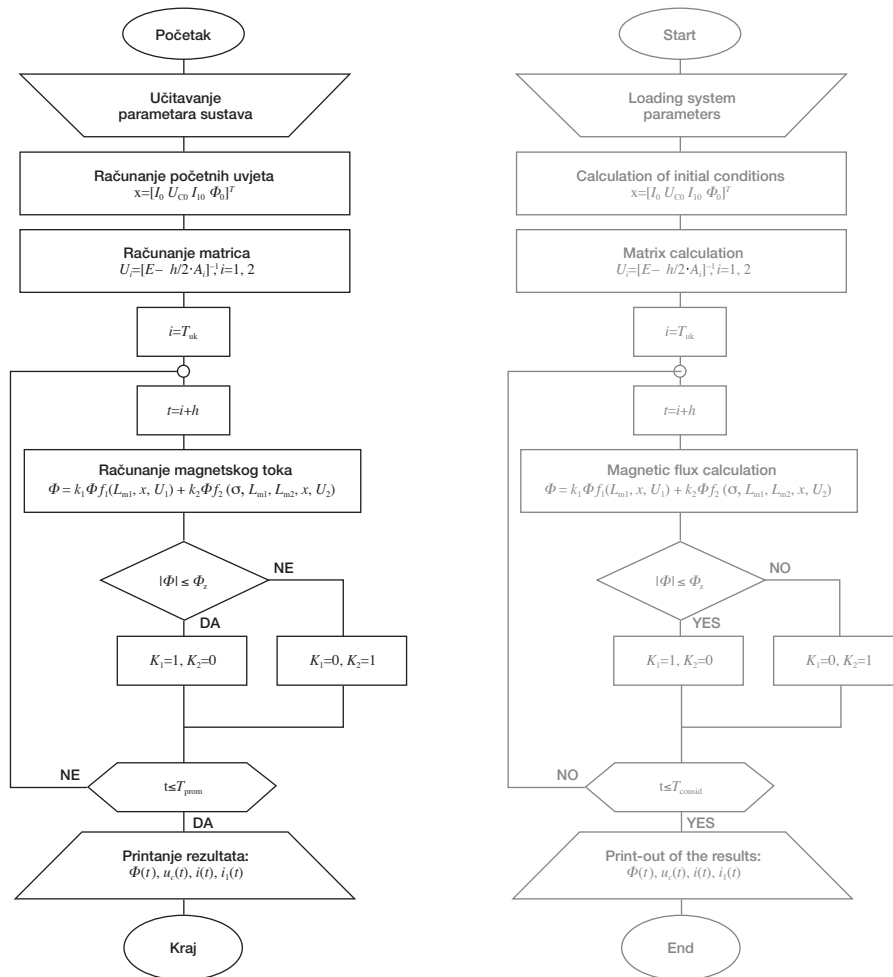
where  $\lambda_i$  denotes all the characteristic values of matrix  $A_i$ . For this example, the step for the unsaturated region would be:

$$h \leq \frac{2 \cdot 4,575 \cdot 10^9}{4,575^2 \cdot 10^{12}} = 4,37 \cdot 10^{-7}. \quad (34)$$

Pri upotrebi implicitnog trapeznog pravila nije potrebno voditi računa o veličini koraka  $h$ . Dakle, moguće je načiniti algoritam, slika 8, koji će prema (32) numerički rješavati sustav diferencijalnih jednadžbi (29) za  $|\Phi| \leq \Phi_z$  i (31) za  $|\Phi| > \Phi_z$ .

When applying the implicit trapezoidal rule, it is not necessary to take the size of step  $h$  into account. Therefore, it is possible to develop an algorithm, Figure 8, which according to (32) will numerically solve the system of differential equations (29) for  $|\Phi| \leq \Phi_z$  and (31) for  $|\Phi| > \Phi_z$ .

**Slika 8**  
 Razvijeni algoritam,  
 potpuni model  
**Figure 8**  
 Developed algorithm,  
 complete model



## 5 TEST PRIMJER

Programi dobiveni prema algoritmima sa slika 6 i 8 testirani su na MATLAB/Simulink/Power System Blockset (PSB), dio MATLAB-a za elektromagnet-ske tranzijente [19]. Parametri modela preuzeti su iz [8]. Rezultati simulacija dani su na slikama 9 i 10.

## 5 TEST EXAMPLE

The programs obtained according to the algorithms from Figures 6 and 8 have been tested using the MATLAB/Simulink/Power System Blockset (PSB), a part of MATLAB for electromagnetic transients [19]. The model parameters were taken from [8]. The simulation results are presented in Figures 9 and 10.

**Slika 9**

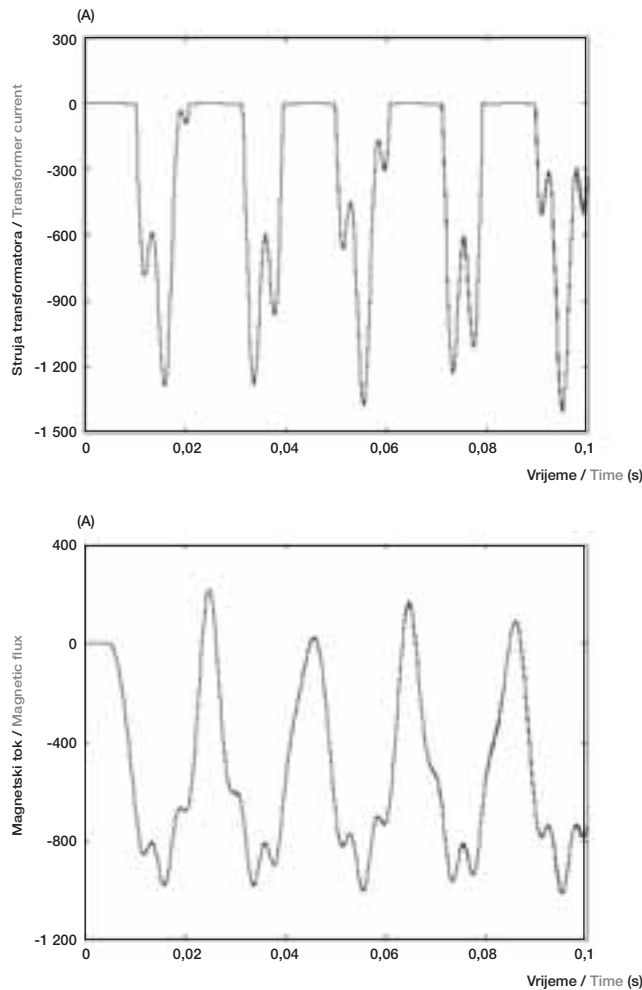
Pojednostavljeni model,

$$T_{uk} = 5 \text{ ms}, \Phi_{rem} = 0$$

Figure 9

Simplified model,

$$T_{uk} = 5 \text{ ms}, \Phi_{rem} = 0$$



Slika 9 pokazuje rezultate razvijenog algoritma koji uzima u obzir pojednostavljeni model uklapanja transformatora, bez prigušnih elemenata, poglavlje 3.

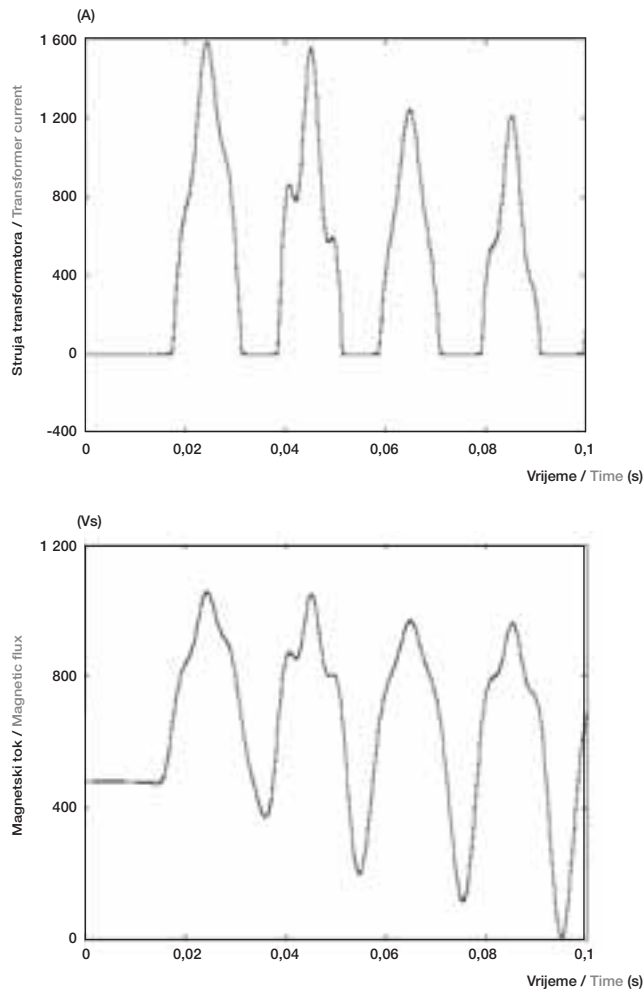
Slika 10 pokazuje rezultate razvijenog algoritma koji uzima u obzir potpuni model uklapanja transformatora, sa svim elementima, poglavlje 4. Na slici 10 je uvažan i remanentni magnetizam transformatora.

Može se zaključiti da se rezultati realiziranih programa u potpunosti podudaraju s programom PSB.

Figure 9 presents the results of the developed algorithm that takes the simplified model of transformer energization into account, without the damping elements, Chapter 3.

Figure 10 presents the results of the developed algorithm that takes the complete model of transformer energization into account, with all the elements, Chapter 4. In Figure 10, the transformer remnant magnetism has also been taken into account.

It can be concluded that the results of the realized programs are in complete agreement with the PSB program.



**Slika 10**  
 Potpuni model,  
 $T_{\text{ak}} = 15 \text{ ms}$ ,  
 $\Phi_{\text{rcm}} = 0,8 \Phi_{\text{nom}}$   
**Figure 10**  
 Complete model,  
 $T_{\text{ak}} = 15 \text{ ms}$ ,  
 $\Phi_{\text{rcm}} = 0,8 \Phi_{\text{nom}}$

## 6 KOMPARACIJA S MJERENJIMA

Razvijeni program se može dalje generalizirati na modeliranje trofaznih transformatora, gdje su krivulje magnetiziranja predstavljene preko konačnog broja pravaca. Tada se vrijednosti struja magnetiziranja, po fazama, računaju iz formule [20]:

## 6 COMPARISON WITH MEASUREMENTS

A developed program can be further generalized for the modeling of three-phase transformers, where the magnetizing curves are represented by a finite number of straight lines. The values of the magnetizing current, according to phases, are then calculated from the following formula [20]:

$$i_{m(l)} = \frac{\Phi_{(l)}}{L_{m(l)}} - \text{sign}(\Phi_{(l)}) \sum_{i=1}^{l-1} \Phi_{(i)} \left( \frac{1}{L_{m(l)}} - \frac{1}{L_{m(l)_i}} \right). \quad (35)$$

U posljednjoj relaciji su krivulje magnetiziranja, po fazama, dane s vektorima:

In the previous expression, the magnetizing curves, according to phases, are represented by vectors:

$$L_{m(j)} = [L_{m(j)_1}, L_{m(j)_2}, \dots, L_{m(j)_N}]^T, \quad (36)$$

$$\Phi_{s(j)} = [\Phi_{s(j)_1}, \Phi_{s(j)_2}, \dots, \Phi_{s(j)_N}]^T, \quad (37)$$

gdje su:

$j = 1, 2, 3$  oznake faza,  
 $N$  = ukupni broj pravaca krivulje magnetiziranja.

Razvijeni algoritam je verificiran kompariranjem izmjerenih i simuliranih struja uklapanja neopterećenog trofaznog transformatora. Parametri trofaznog, trostupnog 2,4 kVA, 0,38/0,5 kV, Y-Y transformatora su:

- napon kratkog spoja  $u_{k\%} = 3 \%$ ,
- djelatni otpor namota po fazi  $R_{tr} = 1,5 \Omega$ ,
- rasipni induktivitet  $L_{tr} = 1 \text{ mH}$ ,
- gubici u jezgri transformatora  $R_m = 4 \text{ 626 } \Omega$ .

Nelinearna krivulja magnetiziranja je predstavljena preko 13 pravaca, [20]. Pri modeliranju trostupnog transformatora uvažena je i nulta reaktancija  $L_0 = 15 \text{ mH}$  [20].

Model transformatora [20], s pridodanom nultom reaktancijom prikazan je na slici 11.

where:

$j = 1, 2, 3$  are phase designations,  
 $N$  = the total number of straight lines of the magnetizing curve.

The developed algorithm is verified by comparison between the measured and simulated inrush currents of the no-load three-phase transformer. The parameters of the three-phase three-legged 2,4 kVA, 0,38/0,5 kV, Y-Y transformer are as follows:

- short-circuit voltage  $u_{k\%} = 3 \%$ ,
- effective resistance per winding phase  $R_{tr} = 1,5 \Omega$ ,
- leakage inductance  $L_{tr} = 1 \text{ mH}$ ,
- iron core losses  $R_m = 4 \text{ 626 } \Omega$ .

A nonlinear magnetizing curve is presented via 13 straight lines [20]. In the modeling of a three-legged transformer, zero reactance  $L_0 = 15 \text{ mH}$  is taken into account [20].

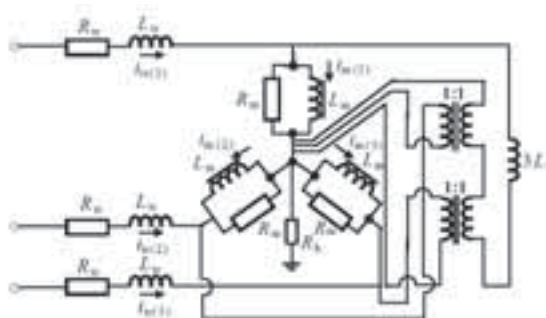
A transformer model [20] with added zero reactance is presented in Figure 11.

#### Slika 11

Model trofaznog, trostupnog transformatora s pridodanom nultom reaktancijom

#### Figure 11

Model of a three-phase, three-legged transformer with added zero reactance



Izvor je modeliran s vektorom elektromotorne sile po fazama:

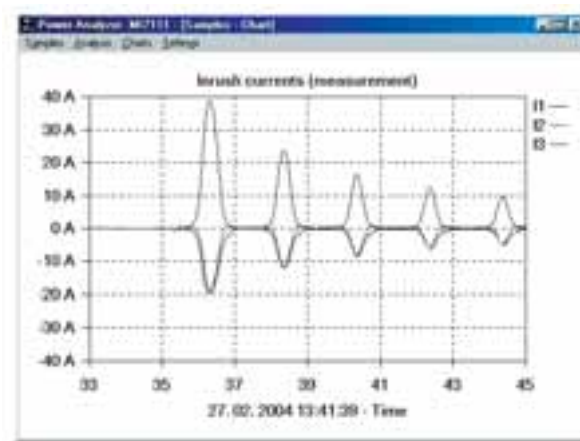
The source is modeled with a vector of the electromotive force for each phase:

$$E = \left[ 311 \cos(\omega t - 34^\circ), 311 \cos(\omega t + 86^\circ), 311 \cos(\omega t + 206^\circ) \right]^T. \quad (38)$$

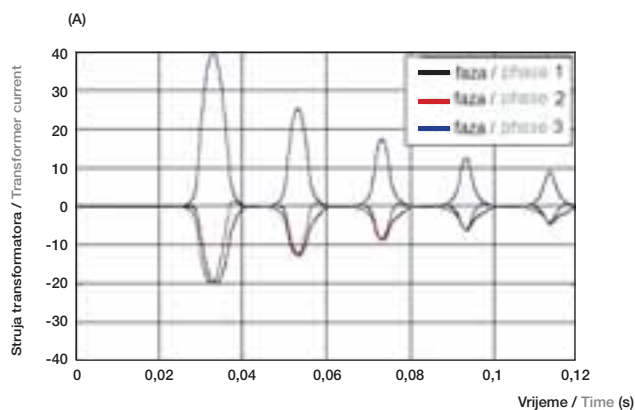
Rezultati mjerenja i simulacija dani su na slici 12.

The measurement and simulation results are presented in Figure 12.

a) Mjerenje / Measurement



b) Simulacija / Simulation



**Slika 12**  
Struje uklapanja trofaznog transformatora: mjerenje i simulacija  
Figure 12  
Inrush current of a three-phase transformer: measurement and simulation

Analizom rezultata mjerenja i simulacija dolazi se do zaključka da maksimalna relativna pogreška, računata po tjemnim vrijednostima struje uklapanja tijekom svakog perioda, iznosi 3,56 %.

Through analysis of the measurement and simulation results, the conclusion is reached that the maximum relative error calculated according to the peak inrush current values during each period amounts to 3,56 %.

## 7 ZAKLJUČAK

U radu je opisan model transformatora primjenjiv u prijelaznim pojavama relativno niskih frekvencija. Za energetske transformatore velikih snaga pokazano je da se krivulja magnetiziranja kvalitativno može predstaviti preko dva pravca. Pokazane su granice upotrebe analitičkih metoda proračuna prijelaznih pojava u transformatorima. Za numeričko rješavanje sustava krutih diferencijalnih jednadžbi iskorišteno je implicitno trapezno pravilo. Razvijeni algoritam je moguće aplicirati u niskofrekvencijskim prijelaznim pojavama kao što su: uklapanje transformatora, ferorezonancija, ispad tereta, kvarovi transformatora itd. Na kraju je pokazan primjer upotrebe realiziranog algoritma na proračun prijelazne pojave uklapanja trofaznog trostepnog transformatora. Kompariranjem mjerenih i simuliranih struja uklapanja trofaznog transformatora ustanovljeno je da se pogreške nalaze u zadovoljavajućim granicama (maksimalno 3,56 %).

## 7 CONCLUSION

A transformer model applicable to relatively low frequency transient phenomena is described in the article. For power transformers with high power ratings, it was demonstrated that the magnetizing curve can be adequately presented with two straight lines. The limits for the use of analytical methods for the calculation of transient phenomena in transformers are presented. The implicit trapezoidal rule was used for the numerical solution of a system of stiff differential equations. The developed algorithm can be applied to low frequency transient phenomena such as transformer energization, ferroresonance, load switch-off, transformer faults etc. An example of the application of the developed algorithm was presented for the calculation of the transients that occur during the energization of a three-phase, three-legged transformer. Through comparison of the measured and simulated inrush currents of the three-phase transformer, it was established that the errors were within acceptable limits (a maximum of 3,56 %).

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