PRORAČUN KVAZISTATIČKOG ELEKTROMAGNETSKOG POLJA SLOŽENIH ELEKTROENERGETSKIH OBJEKATA

COMPUTATION OF QUASISTATIC ELECTROMAGNETIC FIELDS OF COMPLEX ELECTRIC POWER FACILITIES

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Točan proračun niskofrekvencijskog kvazistatičkog elektromagnetskog polja značajan je za projektiranje naprava za prijenos i pretvorbu električne energije. U okviru ovog rada razrađena je teorijska podloga proračuna pomoću integralnih jednadžbi. Postupci rješavanja tih jednadžbi definirani su primjenom metode momenata i metode usklađivanja u točkama. Nepoznata raspodjela gustoće funkcije izvora na tankožičnim elementima je približena polinomima trećeg stupnja. Na pravokutnim plohama približenje nepoznate funkcije izvora realizirano je bilinearnim elementima. Razvijen je program za paralelni proračun elektromagnetskog polja temeljen na opisanoj metodi.
Accurate computation of low-frequency quasistatic electromagnetic fields is important in the design of electricity transmission and conversion devices. This work presents the theoretical groundwork of a computation using integral equations. The procedures for solving these equations are defined by applying the method of moments and the point matching method. The unknown distribution of the source function density on the thin-wire elements is approximated by the third degree polynomials. On rectangular planes the approximation of an unknown source function is achieved by means of bilinear elements. A program has been developed for the parallel electromagnetic field computation based on the described method.

Ključne riječi: integralne jednadžbe; metoda momenata; proračun elektromagnetskog polja
Keywords: integral equations; method of moments; electromagnetic field computation

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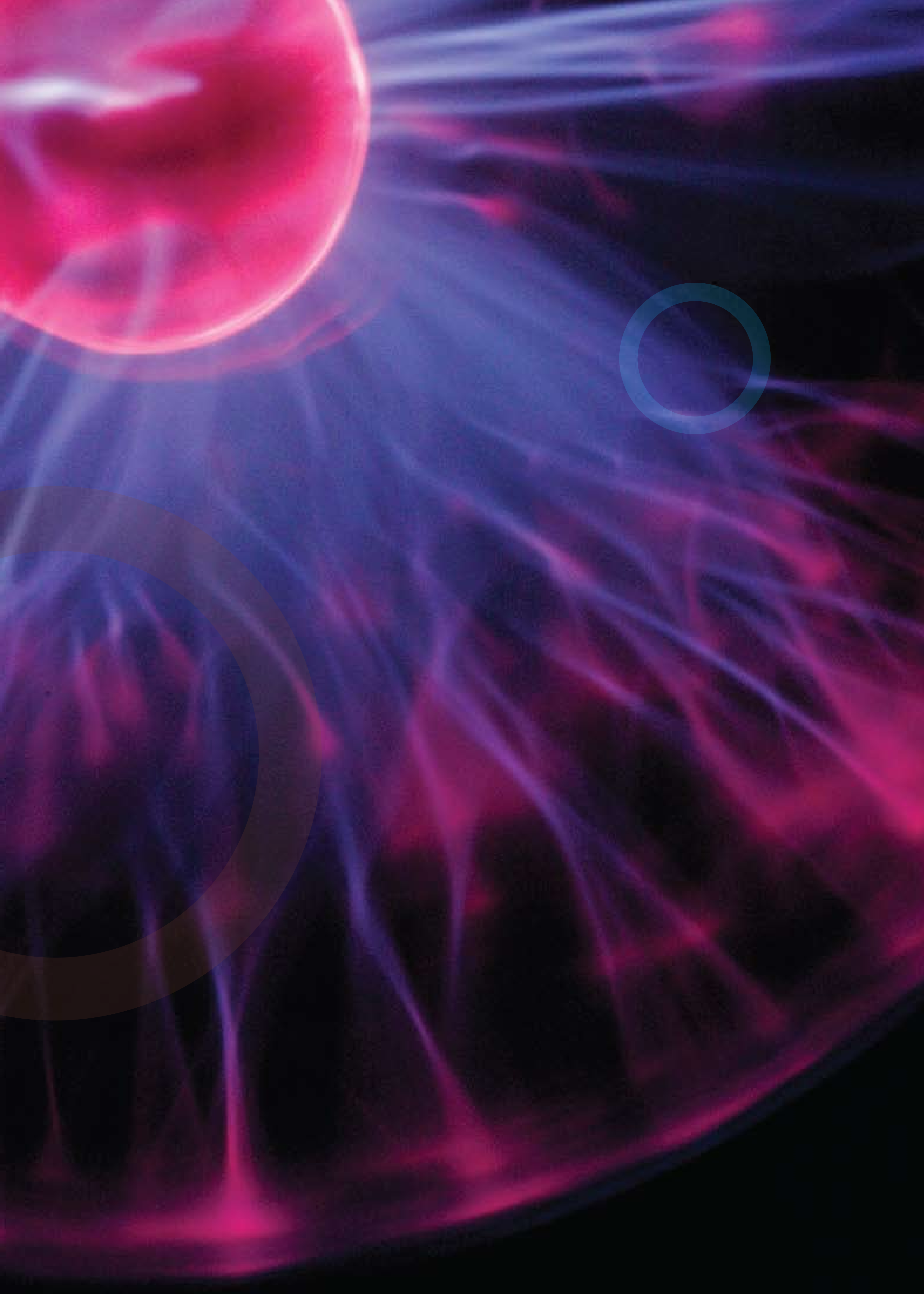


1 UVOD

Elektromagnetska polja industrijske frekvencije tema su brojnih radova i istraživanja u svijetu i kod nas [1], [2], [3] i [4]. Proračun elektromagnetskih polja značajan je za projektiranje naprava za prijenos i pretvorbu električne energije. Nepovoljni utjecaji elektromagnetskog polja mogu se odraziti na djelovanje upravljačkih uređaja, mjernih uređaja, komunikacijskih kanala itd.

Pravilnicima i zakonom su definirane granične vrijednosti polja kojima se smiju izlagati ljudi koji rade u postrojenjima koja su izvor elektromagnetskih polja, kao i stanovništvo koje živi u blizini postrojenja [5]. Zbog toga je prigodom projektiranja postrojenja potrebno provesti proračun elektromagnetskog polja, te tako provjeriti da li je polje unutar dopuštenih graničnih vrijednosti.

U okviru ovog rada razrađena je teorijska podloga proračuna kvazistatičkog elektromagnetskog polja pomoću integralnih jednadžbi. Rješavanje jednadžbi provodi se metodom momenata. Metoda momenata (MoM), koja se često naziva i metoda rubnih elemenata (BEM), često se koristi u inženjerskim primjenama u dinamici fluida, elektromagnetizmu, termodinamici, akustici itd.

Primjena metode momenata ima nekoliko povoljnih i nepovoljnih karakteristika. U metodi momenata potrebno je diskretizirati samo granicu. To bitno smanjuje broj elemenata i olakšava predprocesiranje. Moguć je jednostavan prijenos iz CAD alata. U metodi momenata jednostavno je modelirati probleme koji uključuju otvorene granice proračuna. Značajna negativna osobina metode momenata je potpuno popunjen sustav jednadžbi. Uz primjenu metode kolokacije ovaj sustav je i nesimetričan. Uz primjenu Galerkinove metode sustav je simetričan, ali zahtijeva provođenje još jedne dodatne integracije vrlo složenih funkcija. Potpuno popunjene matrice sustava uzrokuju dugotrajan proračun.

Na temelju teorijske podloge razvijen je računalni program za trodimenzionalni proračun elektromagnetskog polja. Za proračun elektromagnetskog polja metodom momenata potrebno je odrediti unaprijed nepoznatu razdiobu gustoće naboja i struja na površini vodljivih materijala.
Današnji programi [6], često koriste jednostavne procedure za proračun kvazistatičkog elektromagnetskog polja. Jednostavne procedure podrazumijevaju približenje nepoznate funkcije

1 INTRODUCTION

The electromagnetic fields of industrial frequency is the subject of many works and research studies abroad as well as at home [1], [2], [3] and [4]. The electromagnetic field computation is important in the designing of the electricity transmission and conversion devices. The adverse influences of electromagnetic fields can be reflected on the operation of control systems, measuring instruments, communication channels, etc.

Rules and legislative acts lay down the limit field values that the persons working in the facilities which are the source of electromagnetic fields may be exposed to. The same applies to the population living in the vicinity of such facilities [5]. For that reason, when such facilities are designed it is necessary to make an electromagnetic field computation to check if the field fits into the permissible margins.

This work presents a theoretical groundwork for quasistatic electromagnetic field computation by means of integral equations. The equations are being solved by the method of moments. The method of moments (MoM), also referred to as the boundary element method (BEM), is often used in engineering applications in fluid dynamics, thermodynamics, acoustics, etc.

The application of the method of moments has several favorable and unfavorable features. In the MoM only the boundary ought to be discretized. This significantly reduces the number of elements and facilitates pre-processing. A simple transfer from CAD Tools is possible. Problems which include open boundaries can be simply modeled with the MoM. A markedly negative feature of the MoM is a completely filled system of equations. With the use of the collocation method this system is also non-symmetric. With the use of the Galerkin method the system is symmetric, but requires an extra integration process involving highly complex functions. The completely filled system matrices impose a time-demanding computation.

Based on the mentioned theoretical groundwork, a computer program has been developed for the three-dimensional electromagnetic field computation. For the electromagnetic field computation by the method of moments it is necessary to define the charge and current density distribution, not known in advance, on the surface of conductive materials. The present programs [6] often use simple procedures for the quasistatic electromagnetic field computation. Simple procedures imply the approximation of the unknown source function to the constant values on the boundary elements. Such an approach is advantageous in a relatively simple numerical procedure. However, the approximation of the unknown

izvora konstantnim vrijednostima po rubnim elementima. Ovakav pristup ima prednost u relativno jednostavnoj numeričkoj proceduri. Međutim, približenje nepoznate funkcije izvora konstantnim vrijednostima uzrokuje nedovoljnu preciznost. Slaba preciznost može se pokušati kompenzirati vrlo velikim brojem elemenata, što rezultira velikim matricama sustava i praktičnim ograničenjem problema osobito izraženim u 32 – bitnim operativnim sustavima.
Polinomnim približenjem funkcije izvora moguće je točnije približiti funkciju izvora i tako smanjiti broj elemenata u sustavu. Razvijene su funkcije polinomne aproksimacije trećeg reda.

Značajan doprinos ubrzanju proračuna ostvaren je paralelizacijom proračuna. Prednosti korištenja paralelnog proračuna su značajne, a proizlaze iz činjenice da su stupci matrice međusobno nezavisni. Nezavisnost stupaca dovodi do jednostavne raspodjele dijelova proračuna po procesorima. Procesor u radnoj stanici, koja je klijent nikad ne čeka rezultate drugog klijenta i u svakom je trenutku raspoloživ za novo opterećenje od strane servera. Zbog nezavisnosti stupaca u metodi momenata, vrlo je jednostavno i uputno koristiti paralelni proračun.

2 TEORIJSKA PODLOGA PRORAČUNA

Izvori električnog i magnetskog polja u okolini prijenosnih vodova su električne struje i raspodjela naboja na vodičima, kao i inducirane struje i naboji u zemlji i objektima u okolini. Početna točka proračuna elektromagnetskog polja promjenjivog u vremenu su Maxwellove jednadžbe. Na frekvenciji 50 Hz polje je kvazistatičko.

Elektromagnetsko polje se tada sastoji od konzervativne komponente električnog polja koju uzrokuje raspodjela naboja po vodičima i rotacijske komponente uzrokovane protjecanjem struja u vodičima. Složene geometrije potrebno je rješavati trodimenzionalnim proračunom.

S obzirom da se promatra polje u točkama udaljenim od vodiča, može se primijeniti tankožično približenje i vodiče nadomjestiti jednodimenzionalnim linijama.

2.1 Proračun konzervativne komponente jakosti električnog polja

U proračunu konzervativnog električnog polja uzimamo u obzir samo vodiče na poznatim (zadanim) potencijalima. Tlo se uzima u proračun kao vodič na nultom potencijalu pa njegov utjecaj modeliramo tehnikom odslikavanja.

source function to the constant values will result in insufficient precision. An attempt may be made to make up for the poor precision with a very great number of elements, which in turn will result in big system matrices and a practical problem limitation, particularly expressed in 32-bit operating systems. With the polynomial approximation of the source function it is possible to approximate the source function more accurately and thus reduce the number of elements in the system. Third-order polynomial approximation functions have been developed.

An important contribution to a speedier computation has been made through computation parallelization. The advantage of using a parallel computation are significant and come from the fact that the matrix columns are mutually independent. The independence of the columns allows an easy distribution of computation parts among the processors. The processor in the workstation which is a client never waits for the results of another client and is available at any time for a new load from the server. Owing to the independence of the columns in the method of moments, the parallel computation is very easy and recommendable for use.

2 A THEORETICAL COMPUTATION GROUNDWORK

The sources of the electrical and magnetic fields in the environment of transmission lines are electric currents and charge distribution on the conductors, as well as induced currents and charges in earth and buildings in the environment. The starting point of computing an electromagnetic field which is variable in time are Maxwell's equations. At 50 Hz frequency the field is quasistatic.

The electromagnetic field consists then of a conservative electric field component caused by charge distribution on the conductors and a rotational component caused by the flow of electric current in the conductors. Complex geometries should be solved by means of the three-dimensional computation.

Since a field is observed in points distant from the conductors, it is possible to apply the thin-wire approximation and to substitute the conductors by one-dimensional lines.

2.1 Computing the conservative electric field strength component

In the conservative electric field computation only the conductors on known (given) potentials are taken into account. The ground is taken in the computation as a conductor on zero potential, so its influence is modeled by the imaging method.

Fazor skalarnog električnog potencijala $\dot{\varphi}(\boldsymbol{r})$ u nekoj točki $\boldsymbol{r}$ na površini vodiča svezan je s fazorom linijske gustoće naboja $\dot{\lambda}(\boldsymbol{r}')$ u proizvoljnoj točki $\boldsymbol{r}'$ na svim tankožičnim dijelovima integralnom jednadžbom:

The scalar electric potential phasor $\dot{\varphi}(\boldsymbol{r})$ on a point $\boldsymbol{r}$ on the conductor's surface is linked with the lineal charge density phasor $\dot{\lambda}(\boldsymbol{r}')$ in an arbitrary point $\boldsymbol{r}'$ on all thin-wire parts by the following integral equation:

$$\dot{\varphi}(\boldsymbol{r}) - \int_l \frac{\dot{\lambda}(\boldsymbol{r}')\mathrm{d}l}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|} = 0 \,. \tag{1}$$

Za određivanje nepoznate funkcije $\dot{\lambda}(\boldsymbol{r}')$ koristi se metoda momenata [7]. Tanke žice se dijele na segmente konačne veličine Δl_i $(i = 1, ..., N_{\mathrm{seg}})$. Ti segmenti mogu biti dužine, kružni lukovi i dijelovi parabole. Na i – tom konačnom dijelu nepoznatu funkciju $\dot{\lambda}$ se aproksimira s N_{B} temeljnih funkcija t_k kao:

For defining the unknown function $\dot{\lambda}(\boldsymbol{r}')$ the method of moments is used [7]. Thin wires are divided into finite-size segments Δl_i $(i = 1, ..., N_{\mathrm{seg}})$. These segments may be line segments, circular arches and parabola parts. On the i – th final part the unknown function $\dot{\lambda}$ is approximated with N_{B} of base functions t_k as:

$$\dot{\lambda}_i = \sum_{k=1}^{N_{\mathrm{B}}} K_{ik} \cdot t_k \,. \tag{2}$$

Najjednostavniji pristup sastoji se u korištenju temeljnih funkcija koje su konstante na konačnom dijelu ($N_{\mathrm{B}} = 1$). Takav pristup rezultira diskontinuitetima u približenju linijske gustoće naboja i često nezadovoljavajućom točnošću proračuna. Stoga se pretpostavlja kubna ($N_{\mathrm{B}} = 4$) raspodjela na svakom segmentu. Ako se postavi uvjet derivabilnosti na l onda je ovisnost temeljnih funkcija t_k o bezdimenzionalnom parametru χ $(1 \geq \chi \geq 0)$ definirana kao:

The easiest approach is to use the base functions which are constant on the finite segment ($N_{\mathrm{B}} = 1$). Such an approach results in discontinuities in the approximation of the lineal charge density and in an often unsatisfactory computation accuracy. Hence the cubic ($N_{\mathrm{B}} = 4$) distribution on each segment has been chosen. If the derivability condition is set on l then the dependence of the base functions t_k on the non-dimensional parameter χ $(1 \geq \chi \geq 0)$ is defined as:

$$t_k = \sum_{j=1}^{4} a_{kj} \cdot \chi^{j-1};\ k = 1,2,3,4 \,. \tag{3}$$

Koeficijenti a_{kj} su sadržani u matrici $[A]$:

Coefficients a_{kj} are contained in matrix $[A]$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} . \tag{4}$$

Sada se $\dot{\lambda}$ može izraziti pomoću:

Now $\dot{\lambda}$ can be expressed by means of:

$$\dot{\lambda}_i(s) = \sum_{k=1}^{4} K_{ik} \sum_{j=1}^{4} a_{kj} \cdot \chi^{j-1} \,, \tag{5}$$

gdje je:

where:

$$K_{i1} = \dot{\lambda}_i(0)\,; \quad K_{i2} = \frac{\mathrm{d}\,\dot{\lambda}_i}{\mathrm{d}\,\chi}\Big|_{\chi=0}\,,$$
$$K_{i3} = \dot{\lambda}_i(1)\,; \quad K_{i4} = \frac{\mathrm{d}\,\dot{\lambda}_i}{\mathrm{d}\,\chi}\Big|_{\chi=1}\,. \qquad (6)$$

Supstitucija izraza (5) u (1) rezultira s:

Substitution of expression (5) into (1) results in:

$$\dot{\varphi}(\boldsymbol{r}) = \sum_{i=1}^{N_{\mathrm{seg}}}\left[\sum_{k=1}^{2} K_{ik}\int_{\Delta l_i}\frac{t_k(\boldsymbol{r}')\mathrm{d}\,l}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|} + \sum_{k=1}^{2} K_{i+1,k-2}\int_{\Delta l_i}\frac{t_k(\boldsymbol{r}')\mathrm{d}\,l}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|}\right]. \qquad (7)$$

Sustav linearnih algebarskih jednadžbi za nepoznate kompleksne koeficijente K_{ik} dobiva se usklađivanjem u točkama. Pokusne točke se nalaze na jednoj i dvije trećine svakog segmenta. Linijske se integrale u (7) računa numerički primjenom adaptivnog algoritma koji se temelji na Gauss-Kronrodovim pravilima. Rješenje sustava definira diferencijabilno približenje linijskog naboja $\dot{\lambda}$ na svakom segmentu. Nakon toga, pomoću izraza:

The system of linear algebraic equations for the unknown complex coefficients K_{ik} is obtained by the point matching methods. The collocation points are located on one and two thirds of each segment. The line integrals in (7) are computed numerically using an adaptive scheme based on Gauss-Kronrod rules. The system solution defines the differential approximation of the lineal charge $\dot{\lambda}$ on each segment. After that, by means of the expression:

$$\dot{\boldsymbol{E}}(\boldsymbol{r}) = \sum_{i=1}^{N_{seg}}\left[\sum_{k=1}^{2} K_{ik}\int_{\Delta l_i}\frac{t_k(\boldsymbol{r}')(\boldsymbol{r}-\boldsymbol{r}')\mathrm{d}\,l}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|^3} + \sum_{k=3}^{4} K_{i+1,k-2}\int_{\Delta l_i}\frac{t_k(\boldsymbol{r}')(\boldsymbol{r}-\boldsymbol{r}')\mathrm{d}\,l}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|^3}\right], \qquad (8)$$

može se izračunati vektor-fazor konzervativne komponente jakosti električnog polja u točki $\boldsymbol{r}$.

it is possible to compute the vector-phasor of the conservative electric field strength component in point $\boldsymbol{r}$.

2.2 Proračun rotacijske komponente elektromagnetskog polja

Promatra se linearni izotropni poluprostor V_e sa značajkama $\varepsilon = \varepsilon_0$, $\mu = \mu_0, \gamma$ koji se nalazi u slobodnom prostoru V_0. Regija V_e je ograničena s ravninom S_e s jediničnim vektorom okomice $\boldsymbol{n}$. Vanjska pobuda ($\dot{\boldsymbol{B}}_e$, $\dot{\boldsymbol{E}}_e$) koju uzrokuju struje u vodičima računa se pomoću izraza (9) i (10):

2.2 Computing the rotational components of electromagnetic fields

Under observation is the linear isotropic semi-space V_e with characteristics $\varepsilon = \varepsilon_0$, $\mu = \mu_0, \gamma$ situated in the free space V_0. Region V_e is bounded by plane S_e with normal unit vector $\boldsymbol{n}$. The external excitation ($\dot{\boldsymbol{B}}_e$, $\dot{\boldsymbol{E}}_e$) caused by electric currents in the conductors is computed by means of expressions (9) and (10):

$$\dot{\boldsymbol{B}}_{\mathrm{e}} = \int\frac{\mu\cdot\dot{I}\,\mathrm{d}\,\boldsymbol{l}\times(\boldsymbol{r}-\boldsymbol{r}')}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|}\,, \qquad (9)$$

$$\dot{\boldsymbol{E}}_{\mathrm{e}} = \mathrm{j}\omega\int\frac{\mu\cdot\dot{I}\,\mathrm{d}\,\boldsymbol{l}}{4\,\pi|\boldsymbol{r}-\boldsymbol{r}'|}\,. \qquad (10)$$

Elektromagnetsko polje u prostoru V_0 se može izračunati iz jednadžbi (11) i (12):

Electromagnetic field in space V_0 can be computed from the equations (11) and (12):

$$c \cdot \boldsymbol{E} = \boldsymbol{E}_e + \int_S \left[(\boldsymbol{n} \cdot \boldsymbol{E}) \cdot \nabla G_1 + (\boldsymbol{n} \times \boldsymbol{E}) \times \nabla G_1 + (\boldsymbol{n} \times \nabla \times \boldsymbol{E}) G_1 \right] \mathrm{d}S \,, \tag{11}$$

$$c \cdot \boldsymbol{B} = \boldsymbol{B}_e + \int_S \left[(\boldsymbol{n} \cdot \boldsymbol{B}) \cdot \nabla G_1 + (\boldsymbol{n} \times \boldsymbol{B}) \times \nabla G_1 + (\boldsymbol{n} \times \nabla \times \boldsymbol{B}) G_1 \right] \mathrm{d}S \,. \tag{12}$$

Pri tom je $c = 1$ za $\boldsymbol{r} \in V_0$, a $c = \frac{1}{2}$ za $\boldsymbol{r} \in S$

U jednadžbe (11) i (12) se uvodi na granici dva prostora izraz za induciranu električnu struju $\dot{\boldsymbol{K}}_S$, induciranu magnetsku struju $\dot{\boldsymbol{M}}_S$ i električni naboj $\dot{\rho}_e$:

where $c = 1$ za $\boldsymbol{r} \in V_0$, a $c = \frac{1}{2}$ za $\boldsymbol{r} \in S$.

On the boundary of two spaces the expression for induced electric current $\dot{\boldsymbol{K}}_S$, induced magnetic current $\dot{\boldsymbol{M}}_S$ and electric charge $\dot{\rho}_e$ is introduced into equations (11) and (12) :

$$\dot{\boldsymbol{K}}_S(\boldsymbol{r}) = \boldsymbol{n} \times \dot{\boldsymbol{H}}_1(\boldsymbol{r}) \,, \tag{13}$$

$$\dot{\boldsymbol{M}}_S(\boldsymbol{r}) = -\boldsymbol{n} \times \dot{\boldsymbol{E}}_1(\boldsymbol{r}) \,, \tag{14}$$

$$\dot{\rho}_e = \boldsymbol{n} \cdot \boldsymbol{E}_1 \varepsilon_0 \,. \tag{15}$$

Jednadžbe (11) i (12) mogu se tako napisati u obliku:

Equations (11) and (12) can thus be written as follows:

$$\dot{\boldsymbol{B}}_1(\boldsymbol{r}) = \dot{\boldsymbol{B}}_e(\boldsymbol{r}) + \mu_0 \int_S \dot{\boldsymbol{K}}_S \times \nabla' G_1 \, \mathrm{d}S' + \mathrm{j}\frac{1}{\omega} \int_S \nabla \cdot \dot{\boldsymbol{M}}_S \nabla' G_1 \, \mathrm{d}S' \,, \tag{16}$$

$$\dot{\boldsymbol{E}}_1(\boldsymbol{r}) = \dot{\boldsymbol{E}}_e(\boldsymbol{r}) - \mathrm{j}\omega\mu_0 \int_S \dot{\boldsymbol{K}}_S G_1 \, \mathrm{d}S' + \int_S \dot{\boldsymbol{M}}_S \times \nabla' G_1 \, \mathrm{d}S' + \frac{1}{\varepsilon_0} \int_S \dot{\rho}_e \nabla' G_1 \, \mathrm{d}S' \,. \tag{17}$$

Greenova funkcija G_1 definirana je izrazom (18):

Green's function G_1 is defined by expression (18):

$$G_1 = \frac{1}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|} \,. \tag{18}$$

Iz uvjeta na granici V_0 i V_e određuju se inducirane struje i naboji prema izrazima (19), (20) i (21):

From the conditions on the boundary of V_0 and V_e, the induced currents and charges are are obtained using expressions (19), (20) and (21):

$$\begin{gathered} \boldsymbol{n} \times \left(\int_S (\mu_0 G_1 - \mu_0 G_2) \times \dot{\boldsymbol{K}}_S \, \mathrm{d}S' \right) + \boldsymbol{n} \times \left(\int_S \mu_0 \cdot \gamma \cdot G_2 \cdot \dot{\boldsymbol{M}}_S \, \mathrm{d}S' \right) + , \\ \boldsymbol{n} \times \frac{1}{\mathrm{j}\omega} \int_S (\nabla G_1 - \nabla G_2) \nabla' \cdot \dot{\boldsymbol{M}}_S \, \mathrm{d}S' - \mu_0 \dot{\boldsymbol{K}}_S = -\boldsymbol{n} \times \dot{\boldsymbol{B}}_e \end{gathered} \tag{19}$$

$$\boldsymbol{n}\times\left(-\mathrm{j}\omega\mu_0\int_S(G_1-G_2)\dot{\boldsymbol{K}}_\mathrm{S}\,\mathrm{d}S'\right)-\boldsymbol{n}\times\left(\int_S(G_1-G_2)\times\dot{\boldsymbol{M}}_\mathrm{S}\,\mathrm{d}S'\right)-\boldsymbol{n}\times\int_S\nabla G_1\frac{\dot{\rho}_\mathrm{e}}{\varepsilon_0}\,\mathrm{d}S'+\dot{\boldsymbol{M}}_\mathrm{S}=-\boldsymbol{n}\times\dot{\boldsymbol{E}}_\mathrm{e}, \tag{20}$$

$$\boldsymbol{n}\cdot\left(-\mathrm{j}\omega\mu_0\int_S(G_1-G_2)\dot{\boldsymbol{K}}_\mathrm{S}\,\mathrm{d}S'\right)-\boldsymbol{n}\cdot\left(\int_S(G_1-G_2)\times\dot{\boldsymbol{M}}_\mathrm{S}\,\mathrm{d}S'\right)-\boldsymbol{n}\cdot\int_S\nabla G_1\frac{\dot{\rho}_\mathrm{e}}{\varepsilon_0}\,\mathrm{d}S'-\frac{\dot{\rho}_\mathrm{e}}{2\varepsilon_0}=-\boldsymbol{n}\cdot\dot{\boldsymbol{E}}_\mathrm{e}. \tag{21}$$

Greenova funkcija G_2 definirana je izrazom (22):

Green's function G_2 is defined by expression (22):

$$G_2=\frac{\mathrm{e}^{-\mathrm{j}k|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}. \tag{22}$$

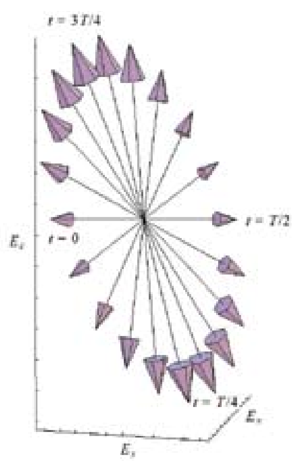
2.3 Prikaz rezultata proračuna

U trodimenzionalnom proračunu vektor jakosti električnog polja je u svakoj točki eliptički polariziran, što znači da vrh vektora $\boldsymbol{E}$ u vremenu opisuje elipsu (slika 1) budući da svaka od triju komponenata općenito ima različitu veličinu i fazni pomak. Za komponente vektora električnog polja vrijedi izraz (23):

2.3 Presentation of computation results

In a three-dimensional computation the electric field strength vector is elliptically polarized in every point, meaning that the terminal point $\boldsymbol{E}$ in time determines the ellipse (Figure 1), because each of the three components generally has a different size and phase shift. Expression (23) applies to the electric field vector components:

$$\begin{aligned}E_x(t)&=E_{x,\max}\cos(\omega t+\varphi_x),\\E_y(t)&=E_{y,\max}\cos(\omega t+\varphi_y),\\E_z(t)&=E_{z,\max}\cos(\omega t+\varphi_z).\end{aligned} \tag{23}$$



Slika 1 — Eliptički polarizirano električno polje
Figure 1 — Elliptically polarized electric field

Efektivna vrijednost vektora električnog i magnetskog polja koristit će se kao rezultat proračuna električnog i magnetskog polja (24):

The effective value of the electric and magnetic filed vectors will be used as a result of the electric and magnetic field computation (24):

$$E = \sqrt{\frac{E_{x,\max}^2 + E2_{y,\max} + E_{z,\max}^2}{2}} \ . \qquad (24)$$

3 PRORAČUN ELEKTROMAGNETSKOG POLJA

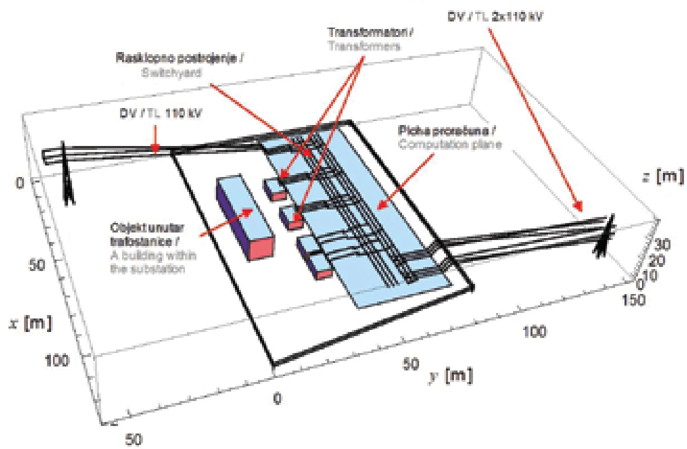
3 ELECTROMAGNETIC FIELD COMPUTATION

Primjer proračuna elektromagnetskog polja proveden je za 110 kV transformatorsku stanicu prikazanu slikom 2.

Jakost električnog polja i magnetska indukcija računaju se na plohi na visini 2m iznad zemlje. Napon na vodičima vanjskog visokonaponskog postrojenja je 110kV. Za proračun rotacijske komponente elektromagnetskog polja koriste se vrijednosti maksimalne struje opterećenja vodiča $I_v = 645$ A, odnosno struje na sabirnicama $I_{sab} = 1\,290$ A.

An example of the electric field computation has been carried out for the 110 kV substation shown in Figure 2.

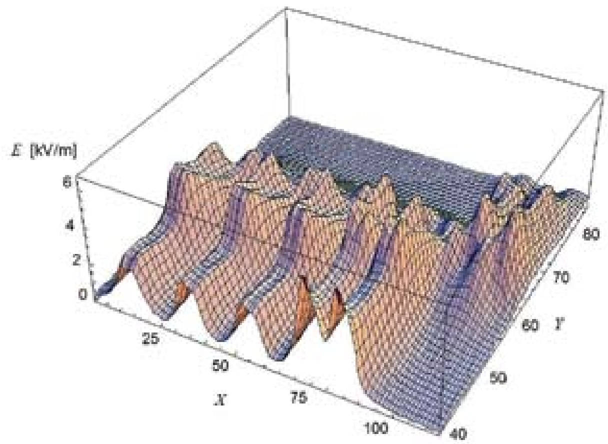
Electric field strength and magnetic induction are computed on a plane at 2 m height above ground level. Voltage on the conductors of an external high-tension facility stands at 110 kV. Used for computing the rotational electromagnetic field component are the values of maximal conductor load current $I_v = 645$ A, or busbar currents $I_{sab} = 1\,290$ A.



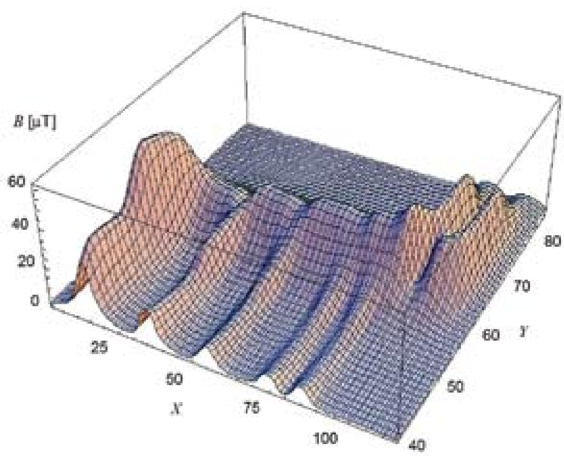
Slika 2 — Model transformatorske stanice
Figure 2 — Substation model

Jakost električnog polja na horizontalnoj plohi proračuna na visini 2 m iznad zemlje unutar postrojenja prikazana je slikom 3.

The electric field strength on the horizontal computation plane at 2 m height above ground level within the facility is shown in Figure 3.



Slika 3 — Jakost električnog polja na visini 2 m iznad zemlje
Figure 3 —Electric field strength at 2 m height above ground



Slika 4 — Magnetska indukcija na visini 2 m iznad zemlje
Figure 4 —Magnetic induction at 2 m height above ground

4 ZAKLJUČAK

Točan proračun niskofrekvencijskih kvazistatičkih elektromagnetskih polja značajan je za projektiranje naprava za prijenos i pretvorbu električne energije. U okviru ovog rada razrađena je teorijska podloga proračuna pomoću integralnih jednadžbi. Postupci rješavanja tih jednadžbi definirani su primjenom metode momenata i tankožičnog približenja vodiča. Nepoznata raspodjela gustoće naboja u proračunu bezvrtložne komponente električnog polja je aproksimirana krivuljama trećeg reda. Vrtložna komponenta elektromagnetskog polja je određena iz zadanih struja uzimajući u obzir utjecaj induciranih vrtložnih struja u vodljivim materijalima. Primjenjivost razvijenih postupaka pokazana je proračunom kvazistatičkog elektromagnetskog polja u primjeru iz prakse.

4 CONCLUSION

A precise computation of low-frequency quasistatic electromagnetic fields is important when it comes to designing the electricity transmission and conversion devices. This work presents a theoretical groundwork for computation by means of integral equations. The procedures for solving these equations are defined by applying the method of moments and the point matching method. The unknown distribution of charge density in the computation of the non-eddy component of an electric field is approximated by third-order curves. The eddy component of an electromagnetic field is defined from given currents taking into account the impact of induced eddy currents in conductive materials. The applicability of the developed procedures is demonstrated on a practical example through the computation of a quasistatic electromagnetic field.

Točnijom aproksimacijom nepoznatih raspodjela gustoće inducirane struje i naboja na granicama vodljivih materijala moguće je točnije odrediti i jakost električnog polja i magnetsku indukciju u prostoru.

A more accurate approximation of unknown density distributions of induced current and charge on the boundaries of conductive materials will allow a more accurate definition of both the electric field strength and the magnetic induction in space.

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