Journal of Energy

VOLUME 64 Number 1–4 | 2015 Special Issue

journal homepage: http://journalofenergy.com/

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DETAILED MATHEMATICAL AND SIMULATION MODEL OF A SYNCHRONOUS GENERATOR

SUMMARY

Synchronous generator theory has been known since the beginning of its use, but the modelling and analysis of synchronous generators is still very existent in the present-day. Modern digital computers enable development of detailed simulation models, thus individual power system elements, including synchronous generators, are represented by the highest degree order models in power system simulation software packages. In this paper, first, a detailed mathematical model of a synchronous generator is described. Then, a simulation model of a synchronous generator developed based on the presented mathematical model. Finally, a transient stability after a short-circuit is simulated using real generator parameters.

Key words: block-diagram model, mathematical model, synchronous generator, short-circuit, transient stability

https://doi.org/10.37798/2015641-4147

1. INTRODUCTION

In electric power system (EPS) simulation software packages today, individual elements can be represented by the models of the highest order of accuracy. By the virtue of state-of-the-art digital computers, in many cases it is not necessary to use simplified and reduced order models based on numerous assumptions anymore. In spite of this, using the most detailed mathematical models does not guarantee the quality and credibility of calculation results. The cause of unsatisfactory results is usually the lack of sufficiently accurate values of parameters on which a certain model is based on. Generally, the more detailed the mathematical model is, the more parameters it requires to be known. As many data for power system calculations (e.g. transient stability, short-circuit, power flow, etc.) are usually hard to obtain, it is clear that it isn't always the best solution to use the most detailed mathematical models. Equipment manufacturers usually provide data about the most of needed parameters, e.g. of synchronous generators, but there are a lot of older generators in the operation today for which it is difficult to determine even the most basic parameters such as synchronous reactance or exciter forced voltage.

Different power system calculations have very different purposes so the demands on accuracy are different as well—from tuning of the protection relays or automatic regulators to analysis of assumed operational scenarios. The issue of detailed modelling, primarily of generators and turbines, and their control systems is especially accentuated in stability calculations. Detailed nonlinear models of generators are described in [1-4]. The most popular is simplified linearized third order model, used by Demello and Concordia [5]. This model is further developed in [6] for small-signal stability analysis. Automatic voltage regulator (AVR) with voltage control loop essentially changes the synchronous generator dynamics. In [7], extended state-space model including the effects of excitation system and generator amortisseurs is used. In this paper the influence of excitation system is not considered and focus is only on generator model. The impact of generator modelling complexity is the subject of many transient stability studies, such as [8-11].

When modelling the synchronous generator, the rest of the EPS is usually replaced with an infinite bus. When researching stability of a generator working in a multi- machine system where the total power is a lot larger than the power of the individual generator (along with a strong grid), only the impact of a short-circuit close to the generator terminals is analysed. As the length of a transient is relatively short (2 s do 5 s), physical properties of the analysed machine have the prevailing impact on the properties of machine swing response.

2. SYNCHRONOUS GENERATOR MODEL

Although the theory of synchronous generator has been known since the beginning of its application, the research of modelling and analysis of synchronous generators is still very much ongoing. Mathematical description of electromechanical systems operation such as synchronous generator generally leads to a system of differential equations which is regularly nonlinear due to the multiplication of state variables. With the increase of computing power, the capabilities for modelling and analysis are increased as well. This has resulted in a large number of models that differ depending on the type of research they are intended for and on the degree of desired accuracy.

There are different approaches when developing a mathematical model and the corresponding simulation model of a synchronous generator. The most common approach is based on general two-reaction theory upon which a three-phase winding of a generator is substituted by one equivalent, fictitious two-phase winding projected onto the direct (*d*) and quadrature (*q*) rotor axis. The field winding is represented as a *d*-axis winding and the reaction of damper winding caused by the eddy currents in the cylindrical rotor is substituted by fictitious windings in *d*-axis and *q*-axis.

3. PARK'S TRANSFORMATION

Mathematical description of a synchronous generator can be significantly simplified with proper variable transformation. One of the possible stator variables (currents, voltages, fluxes) transformation is known as Park's or *d-q* transformation. The number of variables after a transformation generally remains the same and in general case, substitution with new variables should be observed as a completely mathematical operation, thus no physical interpretation of fictitious is necessary. In this case, according to [1], the applied transformation can be physically interpreted because the new variables are obtained by projecting the real variables onto the three axes (direct, quadrature and stationary):

$$\mathbf{i}_{0dq} = \mathbf{P}\,\mathbf{i}_{abc} \tag{1}$$

$$\mathbf{i}_{0dq} = \begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} \quad \mathbf{i}_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2}$$

$$\mathbf{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\vartheta & \cos\left(\vartheta - \frac{2\pi}{3}\right) & \cos\left(\vartheta + \frac{2\pi}{3}\right) \\ \sin\vartheta & \sin\left(\vartheta - \frac{2\pi}{3}\right) & \sin\left(\vartheta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{3}$$

Current i_d can be imagined as a current through a fictitious winding which rotates with the same speed as rotor windings and has such position that its axis always aligns with the field winding axis. The magnitude of current in this fictitious winding will be such that it will induce a magnetomotive force in the d-axis equal to the sum of magnetomotive forces in real phase windings. The current i_q can be imagined in the same way, but the difference is that the axis of the fictitious winding aligns with the neutral axis of the rotor. Current i_0 is identical to the zero-sequence current component and it exists only when the sum of phase currents is different than zero. Zero-sequence is not considered in the generator analysis so the two-reaction representation is simplified which facilitates the setting of generator equations.

Park's transformation is unique, thus an inverse transformation $\mathbf{P}^{-1}$ exists as well, defined as:

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\,\mathbf{i}_{0dq} \tag{4}$$

$$\mathbf{P}^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \cos\vartheta & \sin\vartheta \\ \frac{1}{\sqrt{2}} & \cos\left(\vartheta - \frac{2\pi}{3}\right) & \sin\left(\vartheta - \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \cos\left(\vartheta + \frac{2\pi}{3}\right) & \sin\left(\vartheta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{5}$$

Coefficient $\sqrt{\frac{2}{3}}$ is chosen such that $\mathbf{P}^{-1} = \mathbf{P}^{t}$ which means Park's transformation is orthogonal.

4. VOLTAGE EQUATIONS

Figure 1 shows rotor and stator windings of a three-phase synchronous generator. The considered synchronous generator has three stator windings (*a, b, c*), a field winding (*F*) and two fictitious windings, one in d-axis (*D*) and on in q-axis (*Q*) which substitute the reaction of damper windings or dampening caused by eddy currents in a cylindrical rotor. These six windings are magnetically linked, and flux linkages are a function of the rotor position.

Voltage equations for these six linked circuits can be written in a matrix form:

$$
\left[\begin{array}{c} v_a \\ v_b \\ v_c \\ \hline -v_F \\ -v_D=0 \\ -v_Q=0 \end{array}\right] = \left[\begin{array}{ccc:ccc} r & 0 & 0 & & & \\ 0 & r & 0 & & \mathbf{0} & \\ 0 & 0 & r & & & \\ \hdashline & & & r_F & 0 & 0 \\ & \mathbf{0} & & 0 & r_D & 0 \\ & & & 0 & 0 & r_Q \end{array}\right] \left[\begin{array}{c} i_a \\ i_b \\ i_c \\ \hline i_F \\ i_D \\ i_Q \end{array}\right] - \left[\begin{array}{c} \dot{\psi}_a \\ \dot{\psi}_b \\ \dot{\psi}_c \\ \hline \dot{\psi}_F \\ \dot{\psi}_D \\ \dot{\psi}_Q \end{array}\right] + \left[\begin{array}{c} \mathbf{v}_n \\ \hline \mathbf{0} \end{array}\right] \tag{6}
$$

where

$$
\mathbf{v}_n = -r_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - L_n \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} \tag{7}
$$

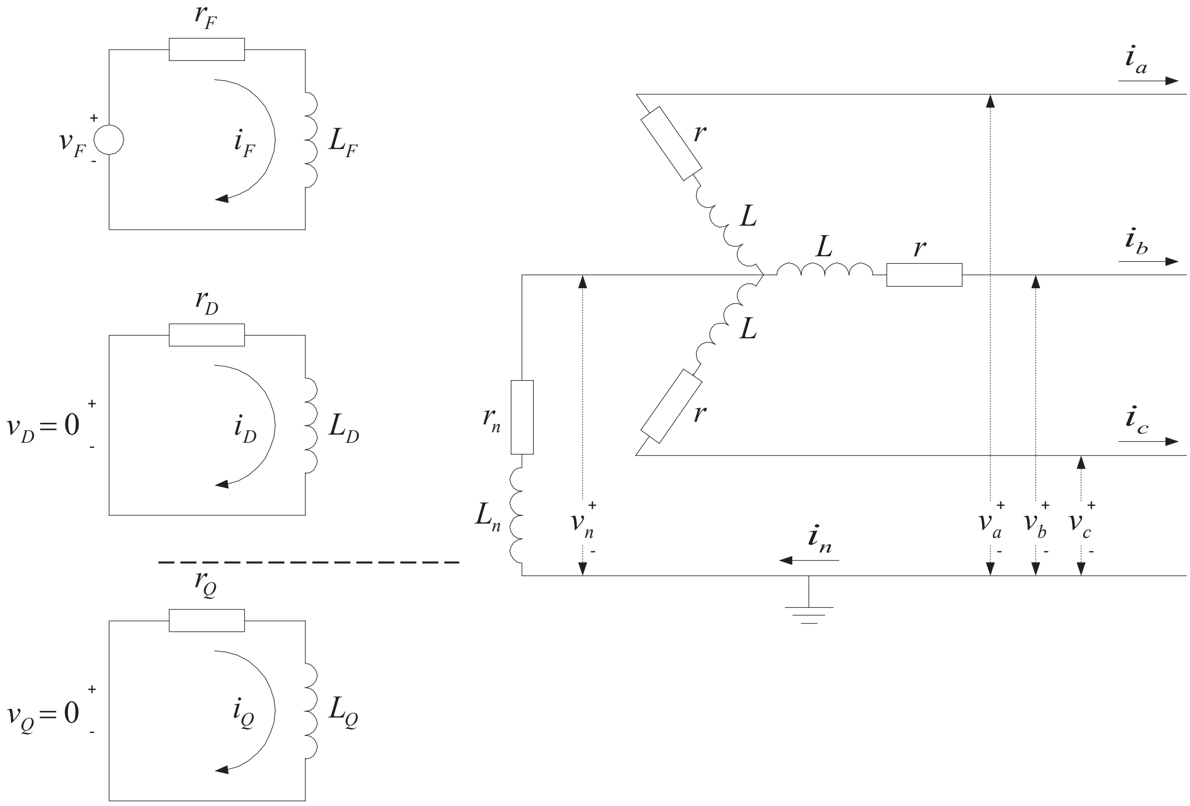

Figure 1. Synchronous generator windings

By applying Park's transformation, (6) becomes

$$\left[\begin{array}{c} v_0 \\ v_d \\ v_q \\ \hline -v_F \\ 0 \\ 0 \end{array}\right] = -\left[\begin{array}{ccc|ccc} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{array}\right] \left[\begin{array}{c} i_0 \\ i_d \\ i_q \\ \hline i_F \\ i_D \\ i_Q \end{array}\right] - \left[\begin{array}{c} \dot{\psi}_0 \\ \dot{\psi}_d \\ \dot{\psi}_q \\ \hline \dot{\psi}_F \\ \dot{\psi}_D \\ \dot{\psi}_Q \end{array}\right] + \left[\begin{array}{c} 0 \\ -\omega\psi_q \\ \omega\psi_d \\ \hline 0 \\ 0 \\ 0 \end{array}\right] \tag{8}$$

By substituting for flux linkages

$$\left[\begin{array}{c} \psi_0 \\ \psi_d \\ \psi_q \\ \hline \psi_F \\ \psi_D \\ \psi_Q \end{array}\right] = \left[\begin{array}{ccc|ccc} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ \hline 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{array}\right] \left[\begin{array}{c} i_0 \\ i_d \\ i_q \\ \hline i_F \\ i_D \\ i_Q \end{array}\right] \tag{9}$$

(8) becomes:

$$\begin{aligned} \left[\begin{array}{c} v_0 \\ v_d \\ v_q \\ \hline -v_F \\ 0 \\ 0 \end{array}\right] = &-\left[\begin{array}{ccc|ccc} r+3r_n & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega L_q & 0 & 0 & k\omega M_Q \\ 0 & -\omega L_d & r & -k\omega M_F & -k\omega M_D & 0 \\ \hline 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{array}\right] \left[\begin{array}{c} i_0 \\ i_d \\ i_q \\ \hline i_F \\ i_D \\ i_Q \end{array}\right] \\ &-\left[\begin{array}{ccc|ccc} L_0+3L_n & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ \hline 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{array}\right] \left[\begin{array}{c} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \hline \dot{i}_F \\ \dot{i}_D \\ \dot{i}_Q \end{array}\right] \end{aligned} \tag{10}$$

As only balanced three-phase systems are usually analysed, the zero-sequence equations are usually omitted. By row-switching in order to group d-axis variables together and q-axis variables together, voltage equations (10) become

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ \hdashline v_q \\ 0 \end{bmatrix} = -\left[\begin{array}{ccc:cc} r & 0 & 0 & \omega L_q & \omega k M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ \hdashline -\omega L_d & -\omega k M_F & -\omega k M_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{array}\right] \begin{bmatrix} i_d \\ i_F \\ i_D \\ \hdashline i_q \\ i_Q \end{bmatrix}$$

$$-\left[\begin{array}{ccc:cc} L_d & kM_F & kM_D & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ \hdashline 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{array}\right] \begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \hdashline \dot{i}_q \\ \dot{i}_Q \end{bmatrix} \tag{11}$$

5. ROTOR SWING EQUATION

Rotor swing equation is usually written in the following form:

$$J \frac{d\omega_m}{dt} = M_m - M_e \tag{12}$$

where J is the moment of inertia (kg·m^2), ω_m is the mechanical angular velocity (rad/s), M_m is the mechanical torque (Nm), M_e the electrical torque (Nm). Difference between mechanical and electrical torque is called an accelerating torque. Equation (12) can be written in terms of power instead of torque:

$$J \frac{d\omega_m}{dt} \omega_m = P_m - P_e \tag{13}$$

Electrical angular velocity is usually used instead of mechanical angular velocity. The relation between mechanical and electrical velocity is given by

$$\omega = p\omega_m \tag{14}$$

where p is the number of pole pairs. It can be shown [1] that by substituting mechanical angular velocity with electrical angular velocity and by introducing per-unit values instead of real values, (12) becomes

$$\frac{2H}{\omega_R} \frac{d\omega}{dt} = M_m - M_e \tag{15}$$

where H is an inertia constant (MWs/MVA), ω_R is the rated electrical speed (rad/s), ω is the electrical angular velocity (rad/s), while mechanical and electrical torque are in per-unit (p.u.). With the assumption that angular velocity ω is approximately constant, the accelerating power is numerically approximately equal to the accelerating torque (p.u.). Thus, the swing equation can be written as

$$\frac{2H}{\omega_R}\frac{d\omega}{dt} \cong P_m - P \tag{16}$$

Rated speed ω_R is equal to

$$\omega_R = 2\pi f_R \tag{17}$$

where f_R is the nominal frequency (Hz), thus (16) can be written as

$$\frac{d\omega}{dt} = \frac{\pi f_R}{H}\left(P_m - P_e\right) \tag{18}$$

The generator swing equation is written in the form of (18). In the case of small disturbances the swing equation could be written as transfer function

$$\frac{\Delta\omega}{\Delta m_m - \Delta m_e} = \frac{1}{2Hs} \tag{19}$$

where is s the Laplace operator [12].

6. ELECTRICAL POWER AND ELECTRICAL TORQUE

Power at the three-phase synchronous generator's terminals is generally calculated as

$$P_e = v_a i_a + v_b i_b + v_c i_c = \mathbf{v}_{abc}^t \mathbf{i}_{abc} \tag{20}$$

By applying Park's transformation on currents and voltages in (20), while keeping in mind that the transformation is orthogonal, the expression for generator power expressed in terms of new voltage and current variables is given as

$$P_e = v_d i_d + v_q i_q + v_0 i_0 \tag{21}$$

As only balanced three-phase systems are usually observed, the expression (21) simplifies to

$$P_e = v_d i_d + v_q i_q \tag{22}$$

By substituting expressions for v_d and v_q from voltage equations the power equation becomes

$$P_e = (i_d\dot{\psi}_d + i_q\dot{\psi}_q) + (i_q\psi_d - i_d\psi_q)\,\omega - r\,(i_d^2 + i_q^2) \tag{23}$$

From this, by using certain assumptions, the simplified expression for an electric torque of a synchronous generators is obtained

$$M_e = i_q \psi_d - i_d \psi_q \tag{24}$$

which is usually used when modelling a synchronous machine.

7. EQUIVALENT CIRCUIT OF A SYNCHRONOUS GENERATOR

By expanding equation (9) for flux linkages, it can be shown that flux linkages of mutual inductances can be written as:

$$\psi_{AD} = i_d (L_d - l_d) + kM_F i_F + kM_D i_D = L_{AD}(i_d + i_F + i_D) \tag{25}$$

$$\psi_{AQ} = i_q (L_q - l_q) + kM_Q i_Q = L_{AQ}(i_q + i_Q) \tag{26}$$

where L_{AD} and L_{AQ} are magnetizing inductances of windings in d and q axes.

$$L_{AD} \cong L_D - l_D = L_F - l_F = L_d - l_d = kM_F = kM_D = M_R \tag{27}$$

$$L_{AQ} \cong L_Q - l_Q = L_q - l_q = kM_Q \tag{28}$$

Expressions (25) and (26) for flux linkages of mutual inductances can be represented by current injection in the magnetizing branch, Figure 2. In order to obtain a complete equivalent circuit, it is necessary to consider voltage equations. From (8), for d-axis windings, the following expressions are obtained:

$$v_d = -r i_d - l \dot{i}_d - L_{AD}(\dot{i}_d + \dot{i}_F + \dot{i}_D) - \omega \psi_q \tag{29}$$

$$-v_F = -r_F i_F - l_F \dot{i}_F - L_{AD}(\dot{i}_d + \dot{i}_F + \dot{i}_D) \tag{30}$$

$$v_D = -r_D i_D - l_D \dot{i}_D - L_{AD}(\dot{i}_d + \dot{i}_F + \dot{i}_D) = 0 \tag{31}$$

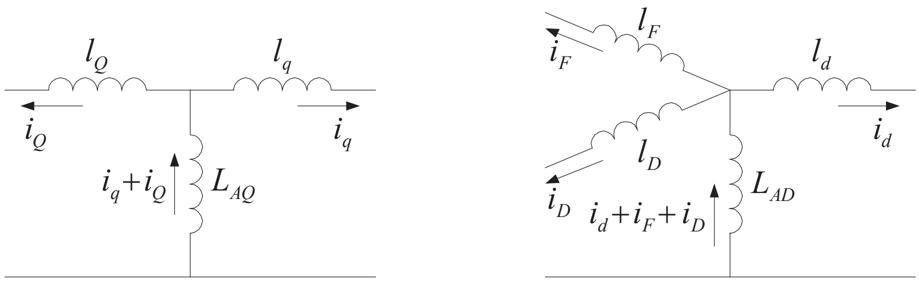

Figure 2. Flux linkages inductances of a synchronous generator

These voltage equations are represented by an equivalent circuit shown in Figure 3. The three circuits (d, F and D) in the d-axis are connected by the mutual inductance L_{AD} through which a sum of currents i_d, i_F and i_D is flowing. A voltage source $\omega\psi_q$ is included in the d-axis stator winding circuit.

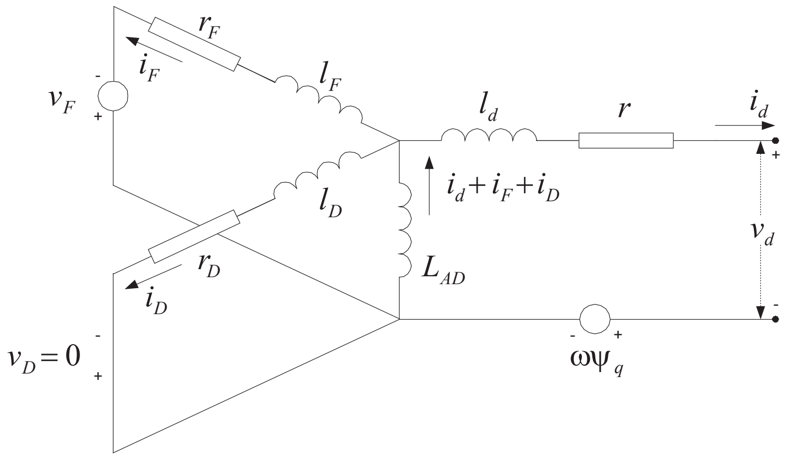

Figure 3. Equivalent circuit of *d*-axis

Voltage equations for *q*-axis windings are as follows:

$$v_q = -r\,i_q - l\,\dot{i}_q - L_{AQ}(\dot{i}_q + \dot{i}_Q) + \omega\psi_d \tag{32}$$

$$v_Q = -r_Q i_Q - l_Q \dot{i}_Q - L_{AQ}(\dot{i}_q + \dot{i}_Q) = 0 \tag{33}$$

and from these equations, the equivalent circuit of *q*-axis is constructed shown in Figure 4. Just like in *d*-axis, the sum of currents also flows through the magnetizing branch and a voltage source $\omega\psi_d$ exists in the *q*-axis winding circuit.

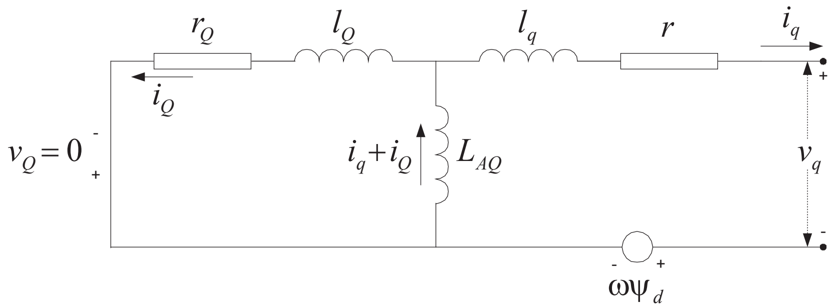

Figure 4. Equivalent circuit of *q*-axis

8. FLUX LINKAGES STATE SPACE MODEL OF A GENERATOR

It can be shown that the following relations between currents and flux linkages result from (9):

d-axis:

$$i_d = \frac{1}{l_d}\left(\psi_d - \psi_{AD}\right) \tag{34}$$

$$i_F = \frac{1}{l_F}\left(\psi_F - \psi_{AD}\right) \tag{35}$$

$$i_D = \frac{1}{l_D}\left(\psi_D - \psi_{AD}\right) \tag{36}$$

where

$$\psi_{AD} = \frac{L_{MD}}{l_d}\psi_d + \frac{L_{MD}}{l_F}\psi_F + \frac{L_{MD}}{l_D}\psi_D \tag{37}$$

with equivalent d-axis inductance defined as:

$$\frac{1}{L_{MD}} = \frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D} \tag{38}$$

q-axis:

$$i_q = \frac{1}{l_d}\left(\psi_q - \psi_{AQ}\right) \tag{39}$$

$$i_Q = \frac{1}{l_Q}\left(\psi_Q - \psi_{AQ}\right) \tag{40}$$

where

$$\psi_{AQ} = \frac{L_{MQ}}{l_q}\psi_q + \frac{L_{MQ}}{l_Q}\psi_Q \tag{41}$$

with equivalent q-axis inductance defined as

$$\frac{1}{L_{MQ}} = \frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} \tag{42}$$

The expressions for flux linkages result from voltage equations (6):

d-axis:

$$\dot{\psi}_d = -\frac{r}{l_d}\psi_d + \frac{r}{l_d}\psi_{AD} - \omega\psi_q - v_d \tag{43}$$

$$\dot{\psi}_F = -\frac{r_F}{l_F}\psi_F + \frac{r_F}{l_F}\psi_{AD} + v_F \tag{44}$$

$$\dot{\psi}_D = -\frac{r_D}{l_D}\psi_D + \frac{r_D}{l_D}\psi_{AD} \tag{45}$$

q-axis:

$$\dot{\psi}_q = -\frac{r}{l_q}\psi_q + \frac{r}{l_q}\psi_{AQ} + \omega\psi_d - v_q \tag{46}$$

$$\dot{\psi}_Q = -\frac{r_Q}{l_Q}\psi_Q + \frac{r_Q}{l_Q}\psi_{AQ} \tag{47}$$

9. LOAD EQUATIONS

Equations (11), (15) and (24) represent a detailed model of a synchronous machine where the currents are state variables. With the assumption that v_F and M_m are known, the aforementioned system of equations does not completely describe the synchronous generator as long as the unknown variables v_d and v_q are not expressed in terms of state variables i_d and i_q. The prerequisite for this is known conditions at the machine's terminals, i.e. the load at the infinite bus must be taken into account as well as the value of impedance between the generator and the grid.

There are different ways to represent the load: constant impedance, constant power, constant current or any of the possible combinations of these three. For generator modelling, the load representation that will define relations between voltages, currents and angular velocity (load angle) obtained by solving the load flow is required. To simplify the generator model analysis, the rest of the electric power system is replaced by an infinite bus, thus the system influence is reduced to an impedance, and magnitude and angle of the voltage phasor at the infinite bus.

For a generator connected to an infinite bus via step-up transformer and a transmission line of equivalent resistance R_e and inductance L_e, the terminal voltage of the generator is calculated as

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} v_{\infty a} \\ v_{\infty b} \\ v_{\infty c} \end{bmatrix} + R_e \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + L_e \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} \tag{48}$$

The infinite bus voltage is a balanced three-phase voltage

$$\mathbf{v}_{\infty abc} = \sqrt{2}\,V_\infty \begin{bmatrix} \cos(\omega_R t + \alpha) \\ \cos(\omega_R t + \alpha - 2\pi/3) \\ \cos(\omega_R t + \alpha + 2\pi/3) \end{bmatrix} \tag{49}$$

where V_∞ is the RMS value of the grid voltage.

It can be shown that by using Park's transformation and (50)

$$\vartheta = \omega_R t + \delta + \pi/2 \tag{50}$$

expression (49) becomes

$$
\mathbf{v}_{\infty 0dq} = \mathbf{P}\,\mathbf{v}_{\infty abc} = \sqrt{3}\,V_{\infty}\begin{bmatrix} 0 \\ -\sin(\delta-\alpha) \\ \cos(\delta-\alpha) \end{bmatrix} \tag{51}
$$

thus, the expression (48) in 0*dq* system is as follows:

$$
\mathbf{v}_{0dq} = V_{\infty}\sqrt{3}\begin{bmatrix} 0 \\ -\sin(\delta-\alpha) \\ \cos(\delta-\alpha) \end{bmatrix} + R_e\mathbf{i}_{0dq} + L_e\dot{\mathbf{i}}_{0dq} - \omega L_e\begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \tag{52}
$$

10. BLOCK DIAGRAM OF A SYNCHRONOUS GENERATOR

To develop a corresponding block element based simulation model from a certain mathematical mode, the mathematical model must be represented by a block diagram. Detailed nonlinear model of a synchronous generator in a block form is shown in figures 5 through 8.

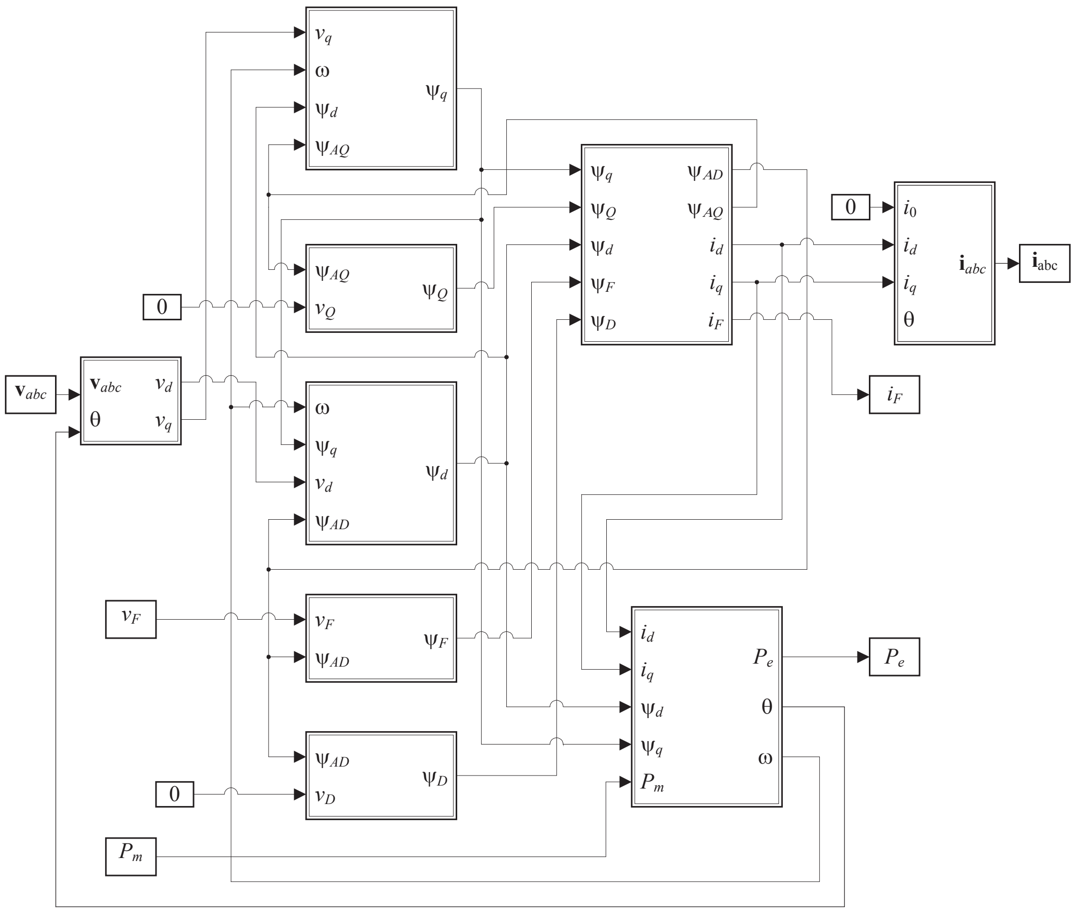

Figure 5. Complete generator system block diagram

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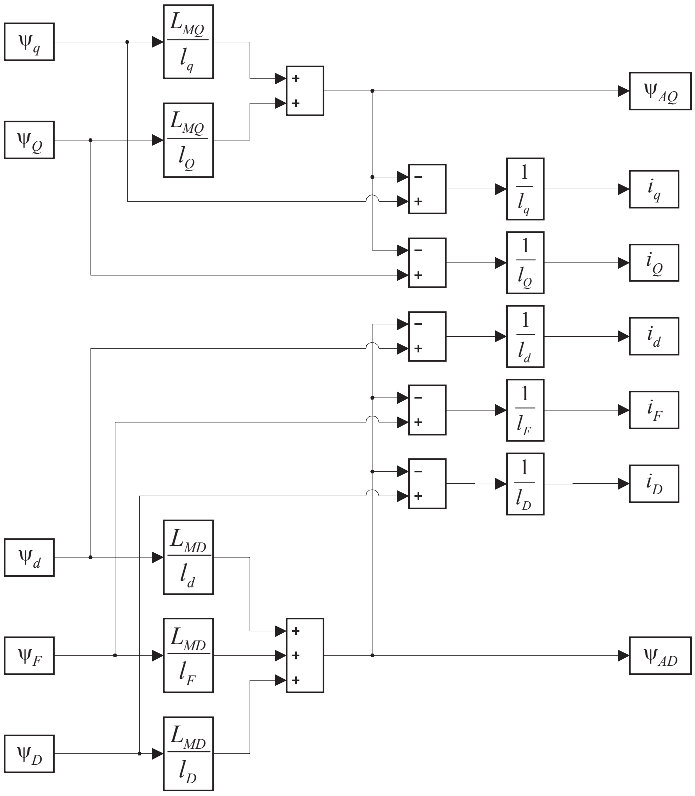


Figure 6. Calculation of currents and flux linkages of mutual inductances

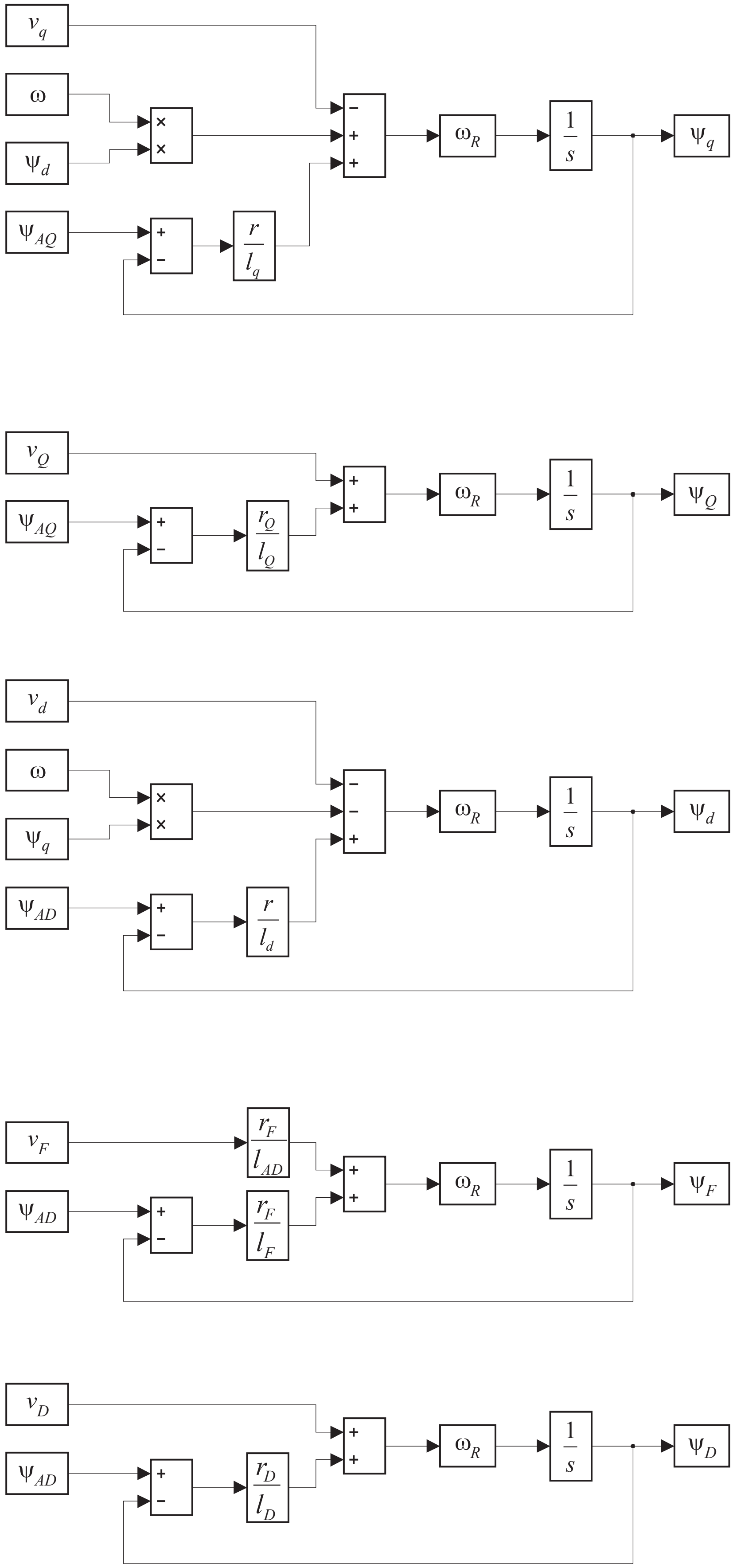


Figure 7. Calculation of flux linkages

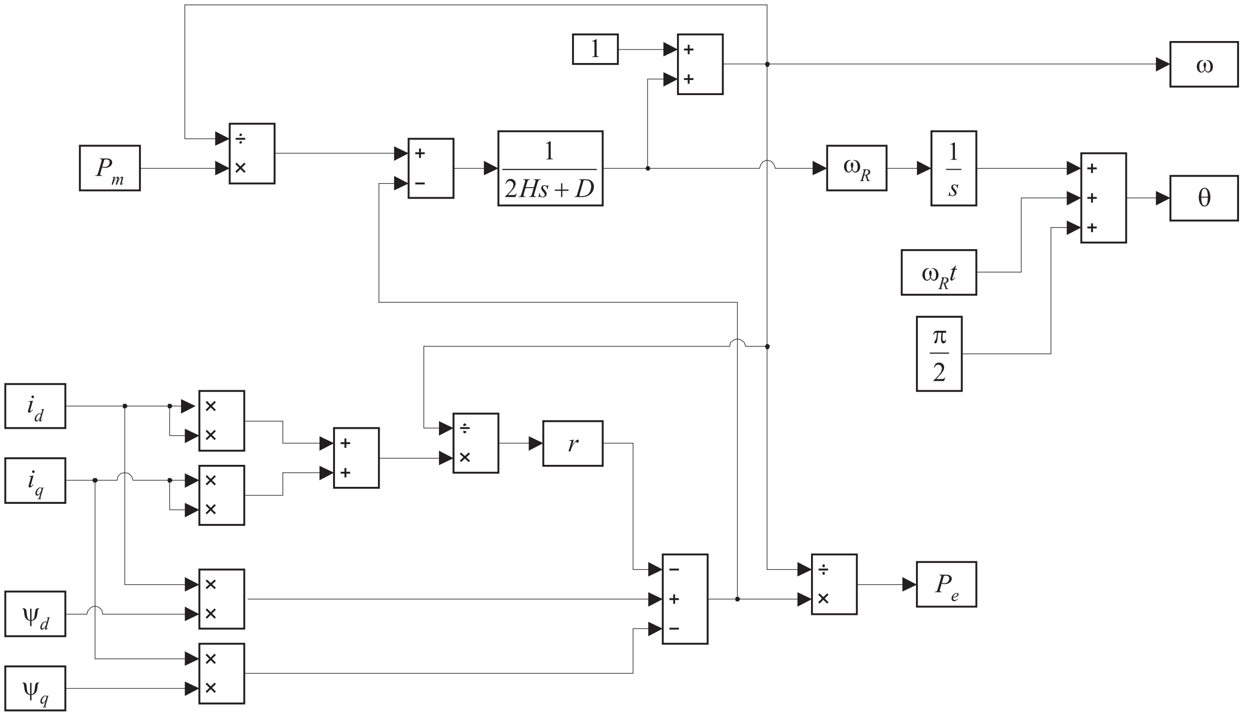


Figure 8. Mechanical part of the generator and electric power calculation

11. SYNCHRONOUS GENERATOR PARAMETERS

Data acquisition necessary for calculations and parameter determination is an important step in the modelling process. Sometimes, acquiring even the most basic generator and corresponding control systems data can present a huge obstacle, especially when dealing with older machines that are still in operation. Thus, generator models with standard parameters are often used, i.e. reactances and time constants identified for the equivalent circuits in the *d* and *q* axis which are given by most generator manufacturers. Standard parameters are being used for the detailed generator model presented in this paper.

12. Standard generator parameters

During a disturbance in the rotor circuits, certain currents are induced under the terms of which some of them diminish more quickly than the others. Thus, the following generator parameters differ:

– subtransient – determine the quickly diminishing components,
– transient – determine the slowly diminishing components,
– synchronous – determine the constant (steady) components

Standard generator parameters are reactances as seen from generator terminals associated with fundamental frequency during steady-state, transient and subtransient states along with corresponding time constants that determine the currents and voltages falloff gradient.

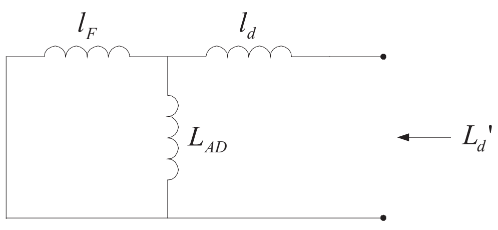
Besides reactances and time constants as standard generator parameters, it is also necessary to know the inertia constant H which determines the dynamic behaviour of the turbine-generator. The value of the inertia constant (MWs/MVA) can be determined using (53)

$$H = \frac{1}{2} \frac{J \omega_m^{\ 2}}{S_n} \tag{53}$$

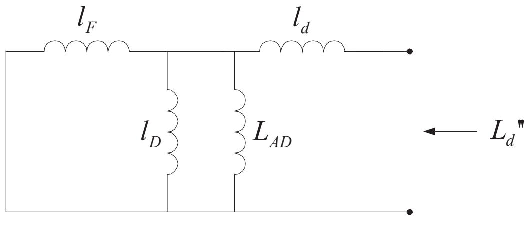
where J is the moment of inertia of the turbine-generator (t·m²), ω_m the (nominal) mechanical speed of the shaft (rad/s), S_n the volt-ampere base of the turbine-generator, usually the nominal apparent power (kVA). Moment of inertia describes the influence of the total rotating mass of the turbine-generator consisting of rotating mass of the turbine and rotating mass of the generator, while the contribution of the water mass must also be considered when dealing with hydroelectric turbines.

13. DETERMINING THE MODEL PARAMETERS FROM STANDARD GENERATOR PARAMETERS

Calculation of rotor mutual inductances is done according to the equivalent circuits (Figure 3 for d-axis, Figure 4 for q-axis) and by utilizing (27) and (28).



(a)



(b)

Figure 9. Equivalent circuit for d-axis inductance: (a) transient, (b) subtransient

d-axis transient inductance according to Figure 9a is given by

$$L_d' = l_d + \frac{L_{AD} l_F}{L_{AD} + l_F} \tag{54}$$

from which the field winding leakage inductance can be expressed as

$$l_F = L_{AD} \frac{L_d' - l_d}{L_d - L_d'} \tag{55}$$

Similarly, according to Figure 9b, d-axis subtransient inductance is given by

$$L_d'' = l_d + \frac{1}{1/L_{AD} + 1/l_D + 1/l_F} \tag{56}$$

from which the d-axis damper winding leakage inductance can be expressed as

$$l_D = L_{AD} l_F \frac{L_d'' - l_d}{L_{AD} l_F - L_F (L_d'' - l_d)} \tag{57}$$

Finally, d-axis damper windings inductance and field winding inductance are given by

$$L_D = L_{AD} + l_D \tag{58}$$

$$L_F = L_{AD} + l_F \tag{59}$$

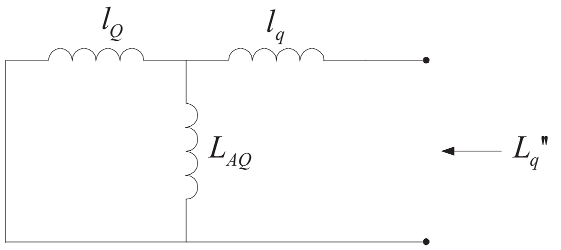


Figure 10. Equivalent circuit for q-axis subtransient inductance

Analogously for the q-axis, from Figure 10 follows:

$$L_q'' = l_q + \frac{L_{AQ} l_Q}{L_{AQ} + l_Q} \tag{60}$$

From which the q-axis damper winding leakage inductance is expressed as

$$l_Q = L_{AQ} \frac{L_q'' - l_q}{L_q - L_q''} \tag{61}$$

and then the q-axis damper winding inductance is given by

$$L_Q = L_{AQ} + l_Q \tag{62}$$

It can be shown that the field winding resistance and the damper windings resistance can be determined from aforementioned reactances by using the following expressions:

$$r_F = \frac{l_F + L_{AD}}{\omega_R T_{d0}'} \tag{63}$$

$$r_D = \frac{l_D + L_d' - l_q}{\omega_R T_{d0}''} \tag{64}$$

$$r_Q = \frac{l_Q + L_{AQ}}{\omega_R T_{q0}''} \tag{65}$$

where the time constants are in (s).

Time constants of short-circuited windings are given by

$$T_d'' = T_{d0}'' \frac{L_d''}{L_d'} \tag{66}$$

$$T_d' = T_{d0}' \frac{L_d'}{L_d} \tag{67}$$

$$T_q'' = T_{q0}'' \frac{L_q''}{L_q} \tag{68}$$

where subscript 0 denotes open circuit time constants.

14. SYNCHRONOUS GENERATOR PARAMETERS

Synchronous generator model described in chapter 2 represents a system of time dependent differential equations. In steady-state, differential equations disappear because all magnitudes are constant. Stability analysis of some system generally begins from a steady state of that system. Then, a disturbance is applied and dynamic behavior is then observed.

Phasor diagrams are usually used to display steady-state relations as shown in Figure 11. Figure 11 displays the phasor diagram for the developed generator model connected to an infinite bus through impedance $R_e + j X_e$.

Steady state can be defined in multiple ways. The most common way is defined by conditions at the generator terminals—voltage, active and reactive power. In this case, the power factor is calculated as

$$\cos\varphi = \frac{P}{\sqrt{P^2 + Q^2}} \tag{69}$$

where P and Q are initial active and reactive power, respectively.

To calculate d-axis and q-axis components of currents and voltages of the generator and of the grid voltage, angles δ, β and φ (see Figure 11) have to be known. δ and β are determined from the phasor diagram and φ is determined from the power factor.

First, generator current is calculated:

$$I = \frac{P}{V\cos\varphi} \tag{70}$$

Then, active and reactive component of generator current are calculated:

$$I_r = I\cos\varphi \quad I_x = -I\sin\varphi \tag{71}$$

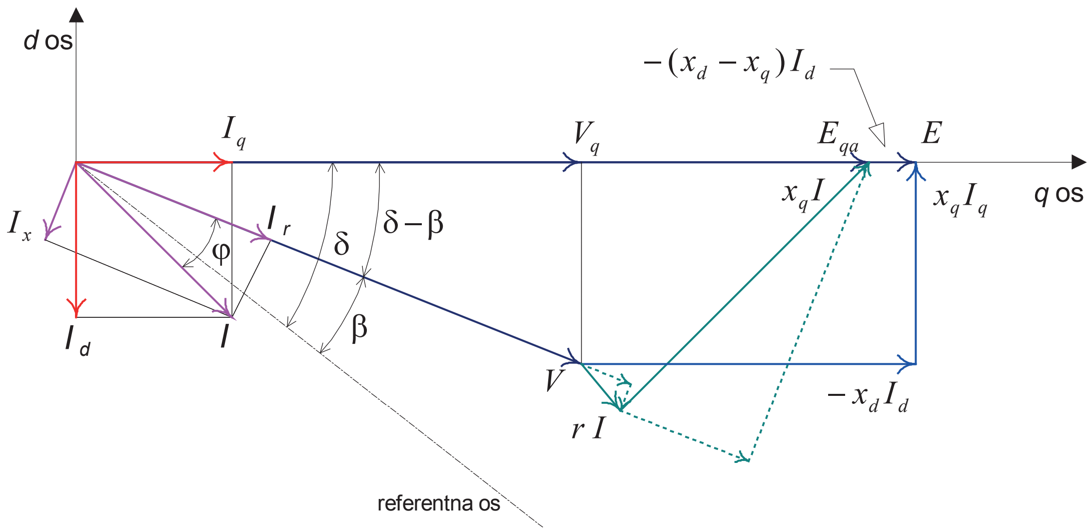


Figure 11. Generator phasor diagram

Angle between q-axis and terminal voltage vector is calculated by

$$\delta - \beta = \arctan\frac{x_q I_r + rI_x}{V + rI_r - x_q I_x} \tag{72}$$

d-axis and q-axis components of generator currents and terminal voltage:

$$I_d = -I\sin(\delta - \beta + \varphi) \quad I_q = I\cos(\delta - \beta + \varphi) \tag{73}$$

$$V_d = -V\sin(\delta - \beta) \quad V_q = V\cos(\delta - \beta) \tag{74}$$

Induced EMF and excitation current:

$$E = V_q + rI_q - x_d I_d \tag{75}$$

$$I_F = \frac{E}{L_{AD}} \tag{76}$$

Flux linkages:

$$\psi_d = L_d I_d + L_{AD} I_F \tag{77}$$

$$\psi_F = L_{AD} I_d + L_F I_F \tag{78}$$

$$\psi_D = (I_d + I_F)\, L_{AD} \tag{79}$$

$$\psi_q = L_q I_q \tag{80}$$

$$\psi_Q = L_{AQ} I_q \tag{81}$$

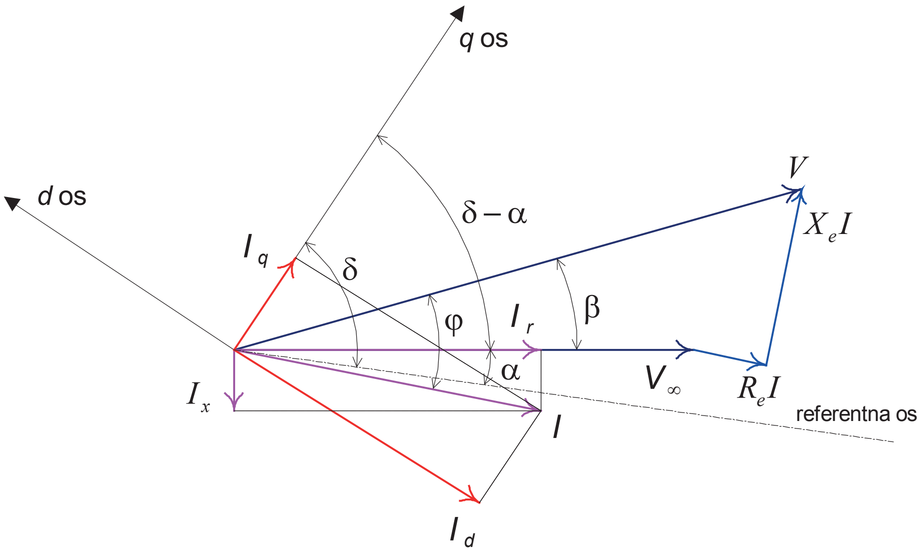


Figure 12. Phasor diagram of generator terminal voltage and grid voltage

Grid voltage vector equation:

$$\overline{V}_\infty = \overline{V} - \overline{Z}_e \overline{I} \tag{82}$$

According to Figure 12, (82) can be expressed as follows:

$$V_\infty \angle(\alpha - \beta) = V - Z_e I\,(\cos(-\varphi) + j\sin(-\varphi)) \tag{83}$$

From (83), grid voltage V_∞ and angle difference $\alpha - \beta$ can be determined. Load angle (angle between grid voltage vector and q-axis) is determined from:

$$\delta - \alpha = (\delta - \beta) - (\alpha - \beta) \tag{84}$$

15. SIMULATION RESULTS

Time-domain simulations have been conducted using the synchronous generator model developed in this paper. Parameters from a real hydroelectric power unit in HPP Dubrava (42 MVA) are used in the simulations. Parameters are shown in Table I.

Table I: Generator parameters of HPP Dubrava

d-axis synchronous reactance	x_d (p.u.)	1.346
q-axis synchronous reactance	x_q (p.u.)	0.940
d-axis transient reactance	x_d' (p.u.)	0.446
d-axis subtransient reactance	x_d''(p.u.)	0.330
q-axis subtransient reactance	x_q''(p.u.)	0.370
Stator leakage reactance	x_l (p.u.)	0.243
d-axis open-circuit transient time constant	$T_{d\,0}'$ (s)	1.660
d-axis open-circuit subtransient time constant	$T_{d\,0}''$ (s)	0.118
q-axis open-circuit subtransient time constant	$T_{q\,0}''$ (s)	0.035
Stator resistance	r (p.u.)	0.006
Inertia constant	H (s)	1.2

Three-phase short-circuit fault at the infinite bus has been simulated as a typical example for different initial conditions.

Current and voltages responses in figure 13 are simulation results for following initial conditions:

generator active power P = 0,75 (p.u.)

generator reactive power Q = 0,25 (p.u.)

generator terminal voltage V = 1 (p.u.) .

Simulations have been also made for different fault durations.

Figure 14 show load angle responses for fault durations of 0.1 s and for fault durations equal to and larger than critical clearing time which is 0.165 s for given scenario.

Model is verified comparing simulated and measured values. Responses of HPP Dubrava generator A on three-phase short circuit in the neighbouring grid (HPP Varaždin) are shown in Figure 15. Simulated and measured responses agree very well, and small differences are probably caused by model parameters which could be calibrated.

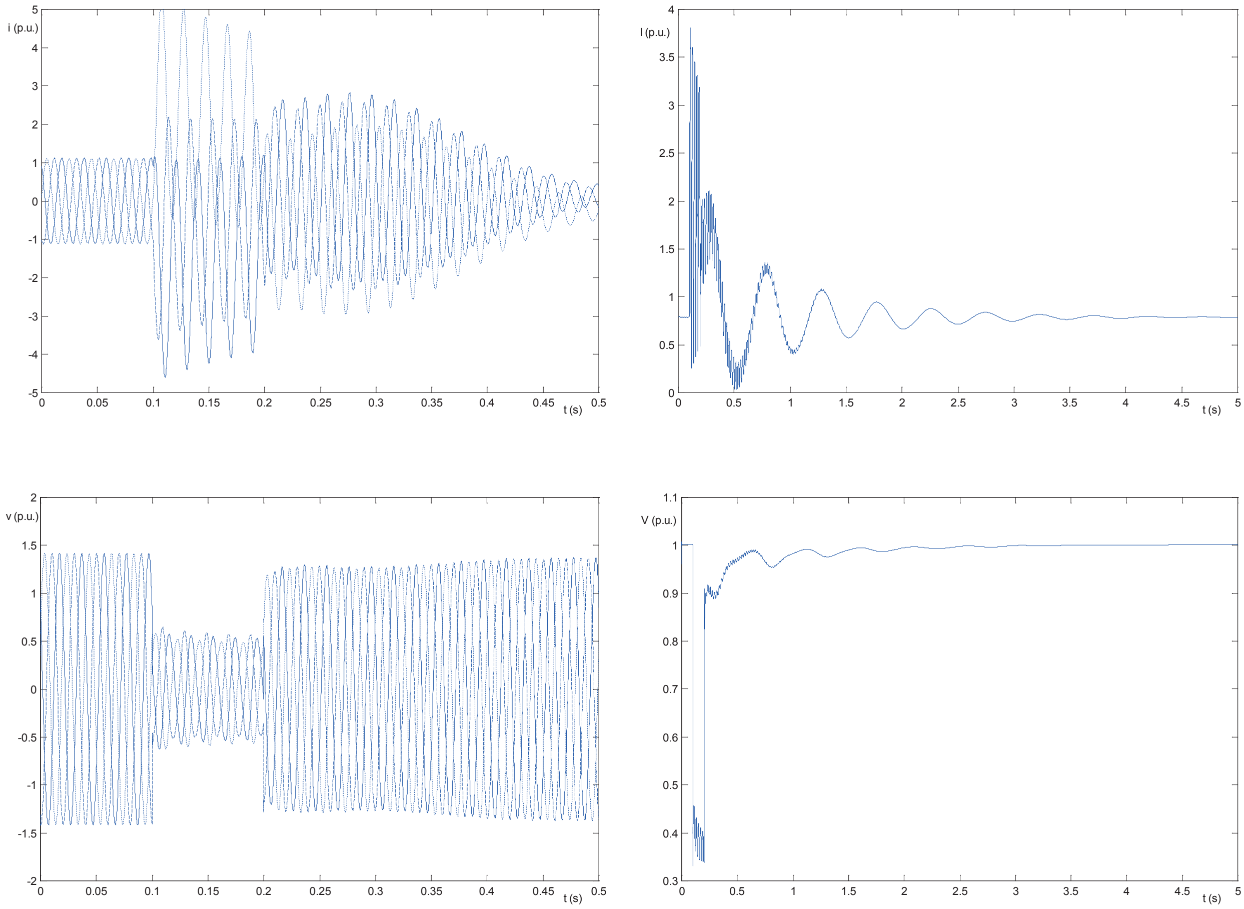


Figure 13. Simulated results – currents and voltages responses (the short-circuit is applied at 0.1 s and removed at 0.2 s)

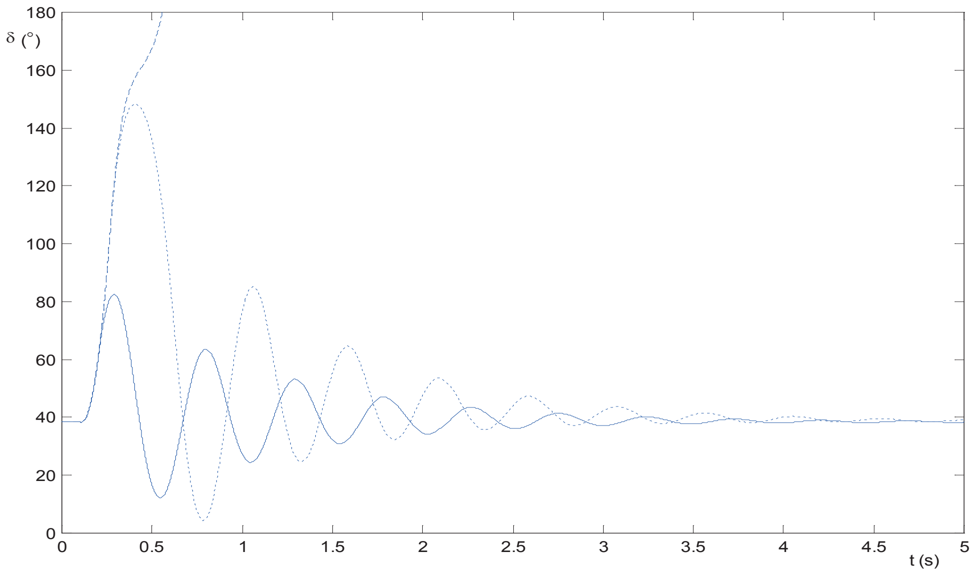


Figure 14. Load angle response for different fault duration (0.1, 0.165 and 0.17 s)

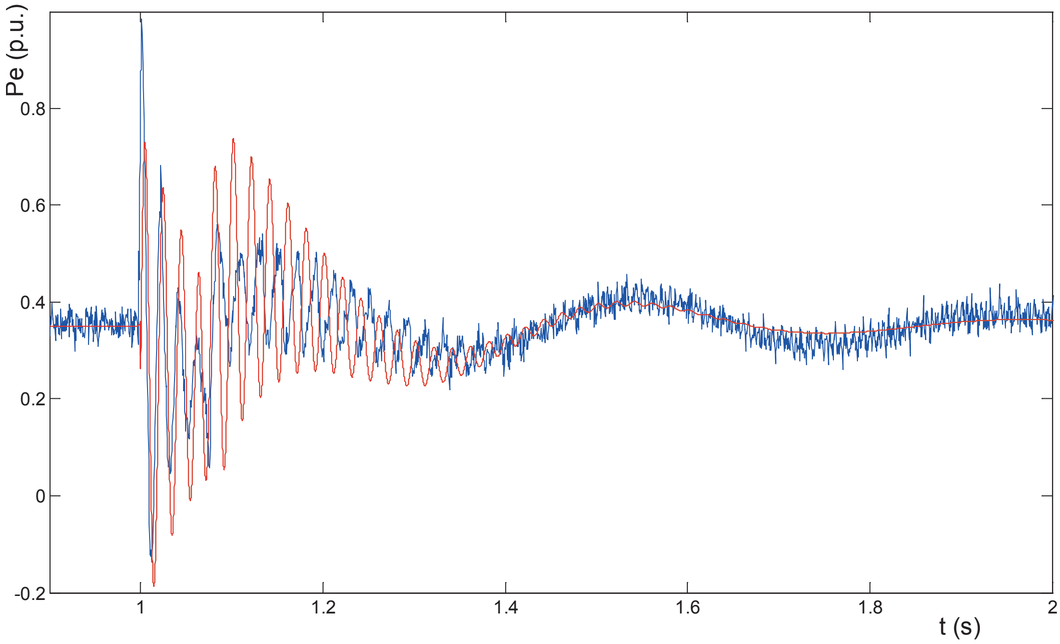


Figure 15. Simulated and measured responses comparison (the short-circuit is applied at 1 s and removed at 1.1 s)

16. CONCLUSION

Thanks to the modern digital simulation systems even the most complex mathematical models can be translated into adequate simulation models. Therefore, high-order models that provide the highest degree of accuracy (with respect to the existing theory) are used more and more for power system elements modelling instead of low-order simplified models for simulating power system operation. The presented mathematical and simulation model of a synchronous generator allows the analysis of all electrical and mechanical units during faults and in different time scales. As an example, in this paper, the generator response to a three-phase short-circuit fault at the infinite bus (most commonly used type of short-circuit fault in stability analysis) have been shown. With minor adjustments, other types of faults can be simulated as well. The change of initial conditions and parameters is simple so different responses can be simulated and compared in order to analyze the impact of different initial conditions and parameters on the dynamic response of a generator. The block diagram model can be easily integrated with other models (foremost, the excitation and voltage control system and turbine with turbine governor systems).

Because of very high accuracy, the described model is used in Power System Laboratory at the Department of Energy and Power Systems, Faculty of Electrical Engineering and Computing, University of Zagreb to compare computer simulations with recorded dynamics of the generator after some switching operations [13].

17. ACKNOWLEDGEMENT

This work was supported by the Croatian Science Foundation under project "FENISG – Flexible Energy Nodes In Low Carbon Smart Grid" (grant no. IP-2013-11-7766).

18. NOMENCLATURE

E	internal EMF induced by excitation current
f_R	nominal frequency (Hz)
H	inertia constant (s)
i_a i_b i_c	generator armature current - phases *a*, *b*, *c*
i_0 i_d i_q	generator current - 0, *d*, *q* system
i_F i_D i_Q	field winding current, *d* and *q* axis damper winding current
I	generator RMS current
I_r I_x	active and reactive component of generator current
I_d I_q	*d* and *q* axis generator current
I_F	excitation current
J	moment of inertia (kg·m^2)
k	mutual inductance coefficient
L_0 L_d L_q	stator winding inductance - 0, *d*, *q* system
L_F L_D L_Q	field winding inductance, *d* and *q* axis damper winding inductance
L_n	generator neutral point grounding inductance
L_d' L_q'	*d* and *q* axis transient inductance
L_q''	*q*-axis subtransient inductance
l_d l_q	stator winding leakage inductance – *d*, *q* components
l_F l_D l_Q	field winding leakage inductance, *d* and *q* axis damper winding leakage inductance
L_{AD} L_{AQ}	*d* and *q* axis winding magnetizing inductance
L_{MD} L_{MQ}	*d* and *q* axis equivalent inductances
L_e	equivalent inductance between the generator and the infinite bus
M_F M_D M_Q	armature winding and field winding mutual inductance, *d* and *q* axis damper winding mutual inductance
M_R	field winding and *d*-axis damper circuit mutual inductance

M_e	electrical torque
M_m	mechanical torque
p	number of pole pairs
P_e	electrical power
P_m	mechanical power
P	(initial) generator active power
Q	(initial) generator reactive power
r	armature winding resistance
r_F r_D r_Q	field winding resistance, d and q axis damper winding resistance
r_n	generator neutral point grounding resistance
R_e	equivalent resistance between the generator and the infinite bus
S_n	generator nominal apparent power (kVA)
t	time (s)
T_{d0}' T_{q0}'	d and q axis open-circuit transient time constant (s)
T_{d0}''	d-axis open-circuit subtransient time constant (s)
T_d' T_q'	d and q axis short-circuit transient time constant (s)
T_d''	d-axis short-circuit subtransient time constant (s)
v_a v_b v_c	generator terminal voltage - phases a, b, c
v_0 v_d v_q	generator terminal voltage – 0, d, q system
v_F v_D v_Q	field winding voltage, d and q axis damper winding voltage
v_n	generator neutral point voltage
$v_{\infty a}$ $v_{\infty b}$ $v_{\infty c}$	infinite bus voltage - phases a, b, c
V_∞	infinite bus RMS voltage
V	generator RMS voltage
V_d V_q	d and q axis generator voltage
x_d x_q	d and q axis synchronous reactance
X_e	equivalent reactance between the generator and the infinite bus
Z_e	equivalent impedance between the generator and the infinite bus
α	infinite bus voltage phase shift (rad)
β	infinite bus voltage and generator voltage phase shift (rad)
δ	q-axis phase shift with respect to the reference axis; load angle (rad)
φ	Phase shift between generator voltage and generator current (rad)
ψ_a ψ_b ψ_c	stator winding flux linkages - phases a, b, c

ψ_0 ψ_d ψ_q	stator winding flux linkages - 0, *d*, *q* system
ψ_F ψ_D ψ_Q	field winding flux linkage, *d* and *q* axis damper winding flux linkage
ψ_{AD} ψ_{AQ}	*d* and *q* axis mutual inductance flux linkages
ϑ	instantaneous generator voltage angle (rad)
ω	angular frequency (rad/s)
ω_m	mechanical angular frequency (rad/s)
ω_R	nominal (synchronous) angular frequency (rad/s)

All magnitudes for which no units have been specified are expressed in per-unit unless specified otherwise in the text.

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