

journal homepage: http://journalofenergy.com

# SLAVKO KRAJCAR PERICA ILAK IVAN RAJŠL

perica.ilak@fer.hr slavko.krajcar@fer.hr ivan.rajšl@fer.hr

University of Zagreb Faculty of Electrical Engineering and Computing

### HYDROPOWER PLANT SIMULTANEOUS BIDING IN ELECTRICITY MARKET AND ANCILLARY SERVICES MARKETS

#### SUMMARY

In a traditional environment, hydropower plant owners seek for minimum cost while in today deregulated environment goal function is profit maximization. Besides electricity only market, power producers can offer their services also in ancillary services markets. By doing so, it is possible to increase expected profit. This paper focuses on simultaneous hydropower plant biding in electricity and ancillary services markets, and purpose is to examine and verify effects of the proposed method on expected profit of hydropower plan owner. A mathematical model based on mixed integer programming approach is used. Head effect is also take into account with price-wise linear performance curves. Prices from real electricity markets and ancillary markets are used, and real hydropower system Lokve-Bayer in Croatia, with focus on hydropower plant Vinodol, is modelled. Obtained results show that there is a notable improvement in expected profit of hydropower plant if presented market bidding approach is used. It is also shown that hydropower plant Vinodol is capable for simultaneous bidding in different power markets.

**Keywords:** multimarket biding; reservoir head effects; mixed integer programming; hydropower plant scheduling;

# 1. NOMENCLATURE

Sets

Т	Set of indices of the steps of the optimization period, planning horizon, $T = \{1, 2,, T_{max}\}, t \in T, T_{max} \in N$
Ι	Set of indices of the reservoirs/plants, $I = \{ "Križ", "Lokve", "Lepenica", "Bajer" \}, i \in I.$
J	Set of indices of the perf. curves $J = \{1\text{-high lvl.}, 2\text{-middle lvl.}, 3\text{-low lvl.}\}, i \in J.$
$U_i$	Set of upstream reservoirs of plant $i$ .
В	Set of indices of the blocks of the piecewise linearization of the unit performance curve $B = \{1, 2, 3\}, b \in B$ .
N	Set of indices of the profit tolerances $N = \{1, 2,, N_{\text{max}}\}, n \in N$ .
Parameters	
M	Conversion factor equal to $3600 \text{ (m}^3 \cdot \text{s} \cdot \text{m}^{-3} \cdot \text{h}^{-1}).$
$X_{\max}(i)$	Maximal content of the reservoir $i$ (m <sup>3</sup> ).
$X_l(i)$	<i>l-th</i> discrete level of the content of the reservoir $i, l \in \{1,2,3,4\}$ (m <sup>3</sup> ).
$X_{\min}(i)$	Minimal content of the reservoir $i$ (m <sup>3</sup> ).
X(i,0)	Initial water content of the reservoir $i$ (m <sup>3</sup> ).
X(i,24)	Final water content of the reservoir $i$ (m <sup>3</sup> ).
W(i,t)	Forecasted natural water inflow of the reservoir $i$ in time step $t$ (m <sup>3</sup> /s).
$\Pi_{\text{spot}}(t)$	Forecasted price of real-time electricity market in time step $t$ (\$/MWh).
$\Pi_e(t)$	Forecasted price of day ahead electricity market in time step $t$ (\$/MWh).
$\Pi_r(t)$	Forecasted price of day ahead regulation market in time step $t$ (\$/MWh).
$\Pi_{sr}(t)$	Forecasted price of day-ahead 10 minute spinning reserve market in time step $t$ (\$/MWh).
$Q_{\min}(i)$	Minimum water discharge of plant $i$ (m <sup>3</sup> /s).
$Q_{\max}(i)$	Maximum water discharge of plant $i$ (m <sup>3</sup> /s).
$Q_{\max}(i, b)$	Maximum water discharge of block $b$ of plant $i$ (m <sup>3</sup> /s).
$B_{\min}(i)$	Ecological minimum of plant $i$ (m <sup>3</sup> /s).
$P0_n(i)$	Minimum power output of plant <i>i</i> for performance curve <i>n</i> , $n \in \{1,2,3,4,5\}$ (MW).
$P_{\max}(i)$	Capacity of plant $i$ (MW).
$\rho_j(i,b)$	Slope of the block $b$ of the performance curve $j$ of plant $i$ (MWs/m <sup>3</sup> ).
$\rho^{-1}(i,t)$	Conversion factor used for converting $(m^3)$ to $(MWh)$ for reservoir <i>i</i> in particular time step <i>t</i> . Meaning calculation of reservoir energy potential $(MWh/m^3)$ in time step <i>t</i> .
$p_{rup}(t)$	Probability of being in Regulation-up state in time step <i>t</i> .
$p_{r m down}(t) \ p_{ m del}(t)$	Probability of being in Regulation-down state in time step $t$ . Probability of spinning reserve to be activated in time step $t$ .

MSR(i)	Maximum sustain ramp rate of plant <i>i</i> (MW/min).
UP(l)	Ramping up limit of plant $i$ (MW/h).
$D\Lambda(l)$	Ramping down limit of plant $i$ (WW/n).
$\Delta_l(i)$	curves of plant $i, l \in \{1,2,3,4\}$ (MW).
$\delta_l(i)$	Difference between minimal values of two neighboring perf. curves of plant $i, l \in \{1,2,3,4\}$ (MW).
Variables	
X(i,t)	Water content of the reservoir $i$ at the end of time step $t$ (m <sup>3</sup> ).
$X_{\text{avg}}(i,t)$	Average water content of the reservoir $i$ in time step $t$ (m <sup>3</sup> ).
q(i,t)	Water discharge of plant <i>i</i> in time step <i>t</i> ( $m^{3}/s$ ).
$q_{\mu}(i,t,b)$	Water discharge of block <i>b</i> of plant <i>i</i> in time step <i>t</i> ( $m^{3}/s$ ).
s(i,t)	Spillage of the reservoir $i$ in time step $t$ (m <sup>3</sup> /s).
$d_k(i,t)$	$0/1$ variable used for discretization of performance curves, $k \in \{1, 2, 3, 4\}$ .
w(i,t,0)	0/1 variable which is equal to 1 if plant <i>i</i> is on-line in time step <i>t</i> .
w(i,t,b)	0/1 variable which is equal to 1 if water discharged by plant $i$ has exceeded block $b$ in time step $t$ .
P(i,t)	Total power output of the performance curve of plant $i$ in time step $t$ (MW).
$P_e(i,t)$	Power output of plant $i$ committed to energy market in time period $t$ (MW).
$P_r(i,t)$	Regulation service capacity of plant $i$ in time period $t$ (MW).
$P_{sr}(i,t)$	10 min spinning reserve of plant $i$ available for increase of output power in time period $t$ (MW).
$P_{tr}(i,t)$	Tertiary reserve of plant $i$ available for increase of output power in time period $t$ (MW).
E(i,t)	Total electricity produced for the energy, regulation and spinning reserve market.
$E_e(i,t)$	Electricity produced for energy market by plant $i$ in time period $t$ (MWh).
$E_r(i,t)$	Electricity produced for regulation service by plant $i$ in time period $t$ (MWh).
$E_{sr}(i,t)$	Electricity produced for spinning reserve service by plant $i$ in time period $t$ (MWh).
$E_{tr}(i,t)$	Electricity produced for tertiary regulation service by plant $i$ in time period $t$ (MWh).

# 2. INTRODUCTION

Water is a scarce resource with uncertain availability. It is therefore complicated to find economically optimal hydropower plant schedule. In a traditional environment, goal function is usually cost minimization [1], [2] and [3] while in the deregulated environment goal function is profit maximization [4] and [5]. In this paper that is based on [6], hydropower plant (HPP) operation in a deregulated environment is considered. Optimization purpose is to find maximum profit with simultaneous bidding on a day-ahead auction market (DAAM) and ancillary services markets. Model is set by *mixed* integer linear programing (MILP) approach with HPP maximum profit as a goal function. In a short-term planning, most of the parameters can be considered as known, and short-term models are called therefore deterministic [7] and [8]. Model in this paper is also deterministic. Some stochastic models are presented in [9], [10] and [11], [12], [13]. Short-term planning also considers effects of water levels in reservoirs on HPP power output. It is, therefore, necessary to model these dependencies between reservoir water levels, turbine discharge and power outputs as described in [14]. This paper is focused on the real hydropower system (HPS), HPS Vinodol, also called HPS Lokve-Bajer because it utilizes water power from Lokvarka and Ličanka basins and some other minor connected basins. Particular attention is given to HPP Vinodol as core building part of HPS Vinodol which data is presented in Figure 1.



Figure 1. The depiction of HPS Vinodol

Data on electrical assets in HPS Vinodol are taken from [15]. Ramp-up and ramp-down speed of power plant is of great importance regarding their role in ancillary services markets. Ramping constraints in thermal power plants are due to the mechanical and thermodynamic stress of turbines, and typical values are 0,03-0,6 p.u.MW/min (0,0005-0,01 p.u.MW/s) [16], [17] and [18]. On the other hand, technical characteristics of HPPs allow fast ramping in both directions. Ramping constraints in hydropower plants are defined by primary regulators, and therefore HPPs have faster ramping in comparison to thermal power plants with typical values of 2,7 p.u.MW/min (0,045 p.u.MW/s) for ramp-up and -3,6 p.u.MW/min (-0,06 p.u.MW/s) for ramp-down [17]. HPP Vinodol can increase power output from zero to nominal power  $P_{max}$  in 22 seconds. It can also decrease power output from nominal power to zero in 17 seconds. Maximum sustain ramp rate for HPP Vinodol in this paper is set to: MSR = 2.7 /-3,6 [p.u.MW/min] and falls into typical value range for

HPP. It is obvious that HPP owners may find some benefits from bidding also on ancillary services markets, in addition to bidding on just electricity markets.

A modelled day ahead electricity market (DAAM) is similar to those deregulated markets such as New England Power Pool, California Market, Australia Electricity Market and New Zealand Electricity Market, where production plan of each power producer or generating company – GENCO is its responsibility in search for maximum profit. GENCO uses PBUC price based unit commitment [19] optimization model for optimal power schedule.

#### 3. PROBLEM DESCRIPTION AND FORMULATION

Model of simultaneous participation of HPP in DAAM and ancillary services markets requires a definition of multilayer problem, namely hydraulic layer, electrical layer and economic layer. Goal function is the profit of HPP Vinodol expressed in (1):

$$\operatorname{Profit}(t) = \sum_{i \in I} [\Pi_e(t) \cdot E_e(i, t) + \Pi_{\operatorname{spot}}(t) \cdot E_r(i, t) + \Pi_r(t) \cdot P_r(i, t) + \Pi_{\operatorname{spot}}(t)$$

$$\cdot E_{sr}(i, t) + \Pi_{sr}(t) \cdot P_{sr}(i, t)]$$

$$(1)$$

where variables and parameters are defined in the nomenclature above.

A mathematical model, analysis and results will be based on a model of real HPS Lokve-Bajer (HPS Vinodol). But since HPP Vinodol is by far largest and most dominant HPP in system optimization criterion is set to be a maximum profit of HPP Vinodol.

#### 4. HPS Lokve-Bajer/ HPS Vinodol

HPS Lokve-Bajer consists of 5 reservoirs (Križ, Lokve, Bajer, Lepenica, Potkoš), two pumped stations (PS) (PS Križ, PS Lič) and 3 HPPs (PHPP Fužine, PHPP Lepenica, HPP Vinodol). Technical characteristics of HPPs and reservoirs are given in table 1. Natural inflows and reservoir seepages are given in table 2.

Reservoir	Volume [hm <sup>3</sup> ]	HPP/PS	Discharge [m <sup>3</sup> /s]	Power [MW]
Križ	0.06	PS Križ	1.1	0.34
Lokve	34.8	PHPP Fužine	10/9*	4.6/4.8*
Bajer	1.32	HPP Vinodol	18.6	94.5
Lepenica	4.26	PHPP Lepenica	6.2/5.3*	1.14/1.25*
Potkoš	0.33	PS Lič	0.45	0.36

Table I Technical characteristics of HPPs and reservoirs, \*turbine/pump

Table II. Natural inflow of reservoirs, \*inflow/seepage

Reservoir	Križ	Lokve	Bajer	Lepenica	Potkoš
Natural inflow	4	4	8/1*	4	4

A mathematical model of HPS Lokve-Bajer consists among other of water balances [14] that describe the relationship between reservoirs in each time step t. HPP Vinodol is modelled by power output curves in more details for already stated reasons while rest HPPs and pumped stations are modelled with a simple linear relationship between turbine discharge and power output.

### 5. Reservoirs

Five reservoirs in HPS Bajer-Lokve are mutually interconnected with pipelines. Reservoir water level (volume) in time step t is determined by reservoir water level (volume) in time step t - 1, natural inflow, turbine discharges and overflows of upstream HPPs connected with a reservoir of interest, and also own turbine discharge and overflow in step t. This linear relationship (2) is called reservoir water balance. In this model time delays are neglected.

$$X(i,t) = X(i,t-1) + M \cdot W(i,t) + M \cdot \sum_{j \in U} [q(j,t) + s(j,t)] - M \cdot [q(i,t) + s(i,t)]$$
  
$$\forall i \in I, \forall t \in T$$

$$(2)$$

# 6. HPPs and pumped stations

Data for the model of HPPs and pumped stations are given in table 3.

Table III. Technical characteristics of HPPs and pumped stations of HPS Lokve-  $$Bajer^{\scriptscriptstyle 1}$$ 

HPP/PS	$Q_t/Q_c (m^3/s)$	Pt/Pc (MW)	H <sub>b</sub> (m)	H <sub>n</sub> (m)
PS Križ	NA/1.1	NA/0.34	8.5	8
PS Lič	NA/0.45	NA/0.36	NA	NA
PHPP Fužine	10.0/9.0	4.6/4.8	NA	37
HPP Vinodol	18.6/NA	94.5/NA	658.5	623

# 7. Power output curves for HPP Vinodol

The power output of HPP Vinodol is modelled by five piecewise linear power output curves. Each curve is used to describe appropriate discrete reservoir part in reservoir Bajer (connected to HPP Vinodol). This nonlinear relationship is called Hill chart [20]. It is set of nonlinear curves called performance curves each defined for the specific water content of reservoir. According to [14] and [20], it is possible to linearize these curves by using binary (0/1) variables and mixed integer linear

<sup>&</sup>lt;sup>1</sup> Source: Hrvatska elektroprivreda

programming approach to precisely model performance curves. The same approach is used in this paper. HPP Vinodol linearized performance curves are shown in figure 2.



Figure 2 The depiction of HPS Vinodol

Activation of appropriate power output curve P(t) in time step t depends on water level (volume) of reservoir X(t) in time step t. It is, therefore, necessary to divide reservoir Bajer into discrete levels as shown in table 4. This activation is presented in [14] and upgraded in [21]. Expressions (3) - (13) are used to model this activation:

Table IV.	Discrete	levels	of reserv	oir	Bajer

Reservoir	Xmin	<b>X</b> 1	X2	X3	X4	Xmax
Bajer	1.0hm <sup>3</sup>	1.1hm <sup>3</sup>	1.20hm <sup>3</sup>	$1.25$ hm $^3$	1.30hm <sup>3</sup>	$1.32 hm^3$

$$X_{avg}(i,t,k) = \frac{X(i,t,k) + X(i,t-1,k)}{2}, \quad \forall i \in I, \forall t \in T, \forall k \in K$$

$$(3)$$

$$\begin{aligned} X_{avg}(i,t,k) &\geq X_1(i) \cdot [d_1(i,t,k) - d_2(i,t,k)] + X_2(i) \cdot [d_2(i,t,k) - d_3(i,t,k)] \\ &+ X_3(i) \cdot [d_3(i,t,k) - d_4(i,t,k)] + X_4(i) \cdot d_4(i,t,k), \\ &\forall i \in I, \forall t \in T, \forall k \in K \end{aligned}$$
(4)

$$\begin{aligned} X_{avg}(i,t,k) &\leq X_{max}(i) \cdot d_4(i,t,k) + X_1(i) \cdot [1 - d_1(i,t,k)] + X_2(i) \\ &\quad \cdot [d_1(i,t,k) - d_2(i,t,k)] + X_3(i) \cdot [d_2(i,t,k) - d_3(i,t,k)] + X_4(i) \\ &\quad \cdot [d_3(i,t,k) - (i,t,k)], \forall i \in I, \forall t \in T, \forall k \in K \end{aligned}$$
(5)

$$d_{1}(i,t,k) \geq d_{2}(i,t,k),$$

$$d_{2}(i,t,k) \geq d_{3}(i,t,k),$$

$$d_{3}(i,t,k) \geq d_{4}(i,t,k), \quad \forall i \in I, \forall t \in T, \forall k \in K$$

$$X(i,t,k) \leq X_{max}(i),$$

$$X(i,t,k) \geq X_{min}(i), \quad \forall i \in I, \forall t \in T, \forall k \in K$$
(6)
$$(7)$$

Each of five power output curves is defined with unique combination of binary decision variables d1, d2, d3, d4 shown in table 5.

Table V. Combinations of binary decision variables  $d_1$ ,  $d_2$ ,  $d_3$ ,  $d_4$ 

Combination	0000	1000	1100	1110	1111
Curve	1	2	3	4	5

For example, when water content X(t) of reservoir is between levels X<sub>1</sub> and X<sub>2</sub>, expressions (3) to (7) sets the binary variables d1, d2, d3, d4 to values 1, 0, 0, 0 and, therefore, activates performance curve 2 using expressions (8) to (13).

$$\begin{split} P(i,t,k) &= P0_{1}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [d_{1}(i,t,k) + d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\leq 0, \\ P(i,t,k) - P0_{1}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [d_{1}(i,t,k) + d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ &\forall i \in I, \forall t \in T, \forall k \in K \\ P(i,t,k) - P0_{2}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [1 - d_{1}(i,t,k) + d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\leq 0, \\ P(i,t,k) - P0_{2}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [1 - d_{1}(i,t,k) + d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ \forall i \in I, \forall t \in T, \forall k \in K \\ P(i,t,k) - P0_{3}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [2 - d_{1}(i,t,k) - d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\leq 0, \\ P(i,t,k) - P0_{3}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [2 - d_{1}(i,t,k) - d_{2}(i,t,k) + d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ \forall i \in I, \forall t \in T, \forall k \in K \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [3 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) + d_{4}(i,t,k)] &\leq 0, \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [3 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) + d_{4}(i,t,k)] &\leq 0, \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [3 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [3 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [3 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) + d_{4}(i,t,k)] &\geq 0, \\ P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ P(i,t,k)$$

$$\begin{split} P(i,t,k) &- P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) - 10 \cdot P_{max}(i) \cdot \\ [4 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) - d_{4}(i,t,k)] &\leq 0, \end{split}$$
(12)  
$$\begin{split} P(i,t,k) - P0_{4}(i) \cdot v(i,t,k) - \sum_{blok=1}^{3} q(i,t,k,blok) \cdot \rho(i,blok) + 10 \cdot P_{max}(i) \cdot \\ [4 - d_{1}(i,t,k) - d_{2}(i,t,k) - d_{3}(i,t,k) - d_{4}(i,t,k)] &\geq 0, \\ \forall i \in I, \forall t \in T, \forall k \in K \end{split}$$
(12)  
$$\begin{split} q(i,t,'1',k) &\leq Q_{1}(i) \cdot v(i,t,k), \\ q(i,t,'1',k) &\geq Q_{1}(i) \cdot w(i,t,'1',k), \\ q(i,t,'2',k) &\leq Q_{2}(i) \cdot w(i,t,'1',k), \\ q(i,t,'2',k) &\geq Q_{2}(i) \cdot w(i,t,'2',k), \\ q(i,t,'3',k) &\leq Q_{3}(i) \cdot w(i,t,'2',k), \end{split}$$
(13)

 $q(i,t,'3',k) \ge Q_3(i) \cdot w(i,t,'3',k), \quad \forall i \in I, \forall t \in T, \forall k \in K$ 

Expressions (8) to (13) define performance curves shown in figure 2. Performance curves are linearized form of Hill chart. Conversion coefficients  $\rho$  [MW / m<sup>3</sup>/s] in (9) to (13) define the efficiency of transformation of water energy (1m<sup>3</sup> in a reservoir) into electrical energy (MWh).

# 8. Ancillary services

Ancillary service that will be provided by HPP in this paper is service of frequency control. That can be achieved by primary, secondary or tertiary regulation for which provision is responsible transmission system operator (TSO) [22]. Criterions that provider must comply with are defined by conditions for connection to transmission grid [23].

# 8.1.1. The primary regulation

Methods of modelling reserve of active power for the primary regulation presented in [24] and [19] are modified to a suite to model of HPS in this paper.

- Regulation-up: In this case, power output must increase. HPP makes a profit based on an available amount of primary reserve  $P_r(t)$  [MW] with a price  $\lambda_r(t)$  [ $\ell$ /MW] in time step t and delivered electricity for regulation  $E_r(t)$  [MWh] with electricity spot price  $\lambda_e(t)$  [ $\ell$ /MWh] while providing regulation in time step t.  $p_{r,up}$  is a probability that HPP will be in the Regulation-up state.
- Regulation-down: In this case, power output must increase. HPP makes a profit based on an available amount of primary reserve  $P_r(t)$  [MW] with a price  $\lambda_r(t)$  [€/MW] in time step t. Due to the decrease in delivered electricity during regulation-down ( $E_r(t)$  [MWh] < 0), HPP does not receive electricity spot price  $\lambda_e(t)$  [€/MWh] for an amount  $E_r(t)$ .  $p_{r,down}$  is a probability that HPP will be in the Regulation-down state.

• No-regulation: In this case, power output does not change. HPP makes a profit based on an available amount of primary reserve  $P_r(t)$  [MW] with a price  $\lambda_r(t)$  [ $\mathcal{C}/MW$ ]. The probability that HPP will be in No-regulation state is  $(1 - p_{r,up} \cdot p_{r,down})$ .

The probability for Regulation-up and Regulation-down are 40% and 35% respectively. This assumption is taken from [25] and presented in table 6.

p <sub>r,up</sub>	<b>p</b> <sub>r,down</sub>	$(1 - p_{r,up} - p_{r,down})$
0.40	0.35	0.25

Table VI. Combinations of binary decision variables  $d_1$ ,  $d_2$ ,  $d_3$ ,  $d_4$ 

In comparison to primary regulation modelling in [24], where mixed integer non-linear programming - MINLP was used, in this paper primary regulation is modelled using mixed integer linear programming – MILP. Piecewise linear performance curves are suitable for the additional description of primary regulation, and it is used in figure 3 (performance curve number 5).



Figure 3 Operational area of HPP while providing primary regulation

The operational area of HPP for producing electricity  $P_e(t)$  becomes narrower if some amount of primary regulation  $P_r(t)$  is provided (Figure 3.). HPP, therefore, must operate below nominal power, and this restricted part is called headroom and defined by (14). Bottom restricted operational area, called foot room, considers that HPP must operate above its minimum power output P0, to be able to provide some regulation-down service. This part is defined by (15). Following expressions define previously stated restricted areas:

$$P_e(t) \le P_{max} - P_r(t) \quad [MW], \forall t \in T$$
(14)

$$P_e(t) \ge P0 + P_r(t) \quad [MW], \ \forall t \in T$$
(15)

Expressions (14) and (15) are actually simplified versions of expressions used in the model. Those more complex expressions for an operational area are following:

$$\begin{aligned} P_e(t) &\leq P_{max} + \Delta_1 \cdot (d_1(t) - 1) + \Delta_2 \cdot (d_2(t) - 1) + \Delta_3 \cdot (d_3(t) - 1) + \Delta_4 \cdot (d_4(t) - (16) \\ 1) - P_r(t) \quad [MW], \forall t \in T \end{aligned}$$

 $P_{e}(t) \ge P0_{5} + \delta_{1} \cdot (d_{1}(t) - 1) + \delta_{2} \cdot (d_{2}(t) - 1) + \delta_{3} \cdot (d_{3}(t) - 1) + \delta_{4} \cdot (d_{4}(t) - (17)) + P_{r}(t) \quad [MW], \forall t \in T$ 

Where symbols  $\Delta_i$ ,  $i \in \{1,2,3,4\}$  denote difference between maximum values of two neighboring power output curves, and symbols  $\delta_i$ ,  $i \in \{1,2,3,4\}$  denote a difference between maximum values of two neighboring power output curves (Figure 4).



Figure 4 Performance curves for HPP Vinodol

Each E(t) represents performance curve P(t) for (expected) power output in [MW], and is defined by (18). Time step t is 1h, and therefore (18) in step t can also be considered as (expected) produced electricity E(t) in [MWh] and is defined by (19).

$$P(t) \coloneqq E(t) = P_e(t) + \left(p_{r,up} - p_{r,down}\right) \cdot P_r(t) [MW] \text{ or}$$
(18)

$$E(t) = E_e(t) + E_r(t) [MWh]$$
<sup>(19)</sup>

Like method presented in [24], top and bottom constraint on primary regulation for HPP are modelled by (20). In this case, a top constraint is equal to a half value of nominal power output of HPP:

$$0 \le P_r(t) \le P_{max}/2 \quad [MW], \ \forall t \in T$$
(20)

8.1.2. Spinning reserve

According to [26], secondary regulation is ancillary service related to secondary control that tries to minimize Area Control Error (ACE). The range of secondary regulation is an interval of active power that is available for remote control by automatic generation control (ACG) within 10 minutes from secondary control activation. The amount of secondary-up or secondary-down regulation is an amount for which active power can be increased or decreased considering an operational state of HPP at the moment of activation of the secondary control. Spinning reserve is modelled similarly to methods presented in [24] and [19]. There are no time constraints regarding the response of the secondary reserve of HPP in [24]. On the other hand, in [19] those temporal constraints are taken into account in the form of maximum sustain ramp rate (MSR) [MW/min] parameter that is defined by the manufacturer. MSR represents maximum stabile ramp rate of power plant and is a very important parameter for frequency control. According to [27] for needs of the primary regulation, an available primary reserve should have following characteristics: the power plant must be able to change power output for 1,5% of nominal power in less than 15 seconds for frequency variations up to 100 mHz and linearly change power output for 3% of nominal power in less than 30 seconds for frequency variations up to 200 mHz. For this paper time constraints, regarding power output changes, in other words, MSR parameter, is set to MSR = 5.67MW/min. According to [19] the secondary regulation available in time step t is:

$$Psr(t) \le \min\{15 \cdot MSR, Pmax - Pe(t)\}\tag{21}$$

and considering also both primary reserve and secondary-up and down reserve:

$$Psr, up(t) \le \min\{15 \cdot MSR, Pmax - Pe(t) - Pr(t)\}$$
(22)

$$Psr, down(t) \le \min\{15 \cdot MSR, Pe(t) - P0 - Pr(t)\}$$
(23)



Figure 5. Footroom and headroom for primary and secondary reserve

Footroom for both primary and secondary reserve is shown in figure 5. The ramp rate of HPP Vinodol (MSR) is several times larger than nominal power, and therefore there are no restrictions on secondary regulation  $(15[min] \cdot MSR[\frac{MW}{min}])$  except required headroom and footroom for the secondary reserve. Ramping constraints for HPP are defined by expressions (24) and (25):

$$Psr, up(t) \le \min\{15 \cdot MSR, Pmax - Pe(t) - Pr(t)\}$$
(24)

$$Psr, down(t) \le \min\{15 \cdot MSR, Pe(t) - P0 - Pr(t)\}$$
(25)

Same assumption. as in the example from [31]. is made. Therefore, HPP Vinodol can change power output very fast, from minimum power P0 to nominal power e  $P_{max}$  within 17 seconds. Expressions (26) and (27), taken from [24], define an amount of the power output changes between two consecutive time steps (hours) that cannot be greater than parameters UP and DR stated in table 7.

$$P(t) - P(t-1) \le UP \tag{26}$$

$$P(t-1) - P(t) \le DR \tag{27}$$

Table VII. Combinations of binary decision variables  $d_1$ ,  $d_2$ ,  $d_3$ ,  $d_4$ 

HPP	MSR Up	MSR Down	15∙MSR up	15·MSR Down	UR	DR
	[p.u.MW/min]	[p.u.MW/min]	[MW/min]	[MW/min]	[MW]	[MW]
Vinodol	2.7	-3.6	94.5	-94.5	94.5	-94.5

Expressions (28) and (29) define an operational area for HPP power output regarding electricity production:

$$P_e(t) \le P_{max} - P_r(t) - P_{sr}(t) \quad [MW], \,\forall t \in T$$
(28)

$$P_e(t) \ge P0 + P_r(t) + P_{sr}(t) \quad [MW], \ \forall t \in T$$
(29)

Expressions (28) and (29) are actually simplified versions of expressions used in model in order to define operational area. Those more complex expressions for operational area are following:

$$\begin{aligned} P_e(t) \le P_{max} + \Delta_1 \cdot (d_1(t) - 1) + \Delta_2 \cdot (d_2(t) - 1) + \Delta_3 \cdot (d_3(t) - 1) + \Delta_4 \cdot (d_4(t) - (30) \\ 1) - P_r(t) - P_{sr}(t) \quad [MW], \, \forall t \in T \end{aligned}$$

$$\begin{aligned} P_e(t) \geq P0_5 + \delta_1 \cdot (d_1(t) - 1) + \delta_2 \cdot (d_2(t) - 1) + \delta_3 \cdot (d_3(t) - 1) + \delta_4 \cdot (d_4(t) - (31) \\ 1) + P_r(t) + P_{sr}(t) \quad [MW], \, \forall t \in T \end{aligned}$$

Where symbols  $\Delta_i$ ,  $i \in \{1,2,3,4\}$  denote a difference between maximum values of two neighboring power output curves, and symbols  $\delta_i$ ,  $i \in \{1,2,3,4\}$  denote difference between maximum values of two neighboring power output curves. When HPP participate in ancillary services market in particular time step t, HPP makes a profit from that market and also from electricity market. If HPP participates in the secondary regulation market following situations can occur:

- HPP also participate in electricity market: in this state, HPP also makes a profit from electricity market besides the secondary reserve market. The probability of this state is  $p_{del}$ .
- HPP does not participate in electricity market: in this state, HPP makes a profit just from secondary reserve market. The probability of this state is  $(1 p_{del})$ .

In order to calculate HPP profit it is necessary to determine amount of electricity that HPP has produced for needs of secondary regulation  $E_{sr}(t)[MWh]$ . Therefore  $p_{del} \cdot E_{sr}(t)$  is used in expression (33) that defines total produced electricity in time step *t*. It is important to note that this is actually expected value.

$$P(t) \coloneqq E(t) = P_e(t) + p_{del} \cdot P_{sr}(t) \quad [MW] \qquad \text{or} \tag{32}$$

$$E(t) = E_e(t) + E_{sr}(t) \ [MWh] \tag{33}$$

Expected power output,  $p_{del} \cdot P_{sr}(t)$ ,  $(p_{del} \cdot E_{sr}(t))$  and water discharge q(t) from HPP Vinodol in time step t are shown in figure 6.





#### 8.1.3. The tertiary (cold) reserve

The tertiary reserve can be divided into fast and slow. The fast tertiary reserve is activated for necessary secondary reserve backup. It is also called minute backup and must mitigate effects of an outage of a largest producing power plant in the system [28]. The slow tertiary reserve is needed for an optimization of power flows and electricity production in the system [29]. The tertiary reserve can also be used for congestion management by the rescheduling of production in one regulation area. The operational area of HPP, that offers tertiary reserve, is shown on figure 5 and defined by expressions (34) and (35) and additionally (36) and (37):

$$P_{tr}(t) \le P_{max} \text{ [MW]}, \forall t \in T$$
 (34)

$$P_{tr}(t) \ge P0 \quad [MW], \ \forall t \in T \tag{35}$$

Expressions (35) and (36) are more complex and are used in this model. They define cold reserve and its range:

$$P_{tr}(t) \le P_{max} + \Delta_1 \cdot (d_1(t) - 1) + \Delta_2 \cdot (d_2(t) - 1) + \Delta_3 \cdot (d_3(t) - 1) + \Delta_4 \cdot (d_4(t) - (36))$$
1) [MW],  $\forall t \in T$ 

 $P_{tr}(t) \ge P0_5 + \delta_1 \cdot (d_1(t) - 1) + \delta_2 \cdot (d_2(t) - 1) + \delta_3 \cdot (d_3(t) - 1) + \delta_4 \cdot (d_4(t) - 1 \quad (37)$ [MW],  $\forall t \in T$ 

To calculate HPP profit, it is necessary to determine an amount of electricity that HPP has produced for needs of the tertiary regulation  $E_{tr}(t)[MWh]$ . Therefore,  $p_{tr,del} \cdot P_{tr}(t)$  is used in expressions (38).

• Probability that HPP will be chosen for tertiary reserve in time step t is  $p_{tr,del}$ . That probability is used to determine expected produced electricity in time step t using expression (38). In this case, HPP makes a profit both from participation on the tertiary reserve market and delivered electricity during provision of tertiary reserve.

$$E_{tr}(t) = p_{tr,del} \cdot P_{tr}(t) \quad [MW]$$
(38)

#### 8.1.4. About electricity market and ancillary services markets

HPP Vinodol is assumed to be price taker. Ancillary services markets modelled in this paper are previously mentioned primary, secondary and tertiary reserves. Due to issues of sequentially performed market clearings of electricity market and ancillary services markets discussed in [30] in this paper simultaneous clearing of all markets is assumed in like presented in [31]. Furthermore, model of market structure is assumed to be based on PBUC [31] (price based unit commitment) approach. Electricity producer makes decision on activating power plan unit according to his risk analysis. In this approach GENCO takes all risk of unit scheduling and commitment. In this paper same GENCO owns power plant units from HPS Lokve-Bajer. Goal function is therefore maximum profit of the HPS considering PBUS approach. In order to make optimal power plant schedule it is required to predict day ahead market prices (for electricity market, primary, secondary and tertiary reserve market) as accurate as possible. Day-ahead power plant schedule is submitted the day before and consists of:

- Vertical bidding curve  $C_{ee}(P(t))$  with point in  $P_{ee}(t)$  for every time step t for electricity market:  $P_{ee}(t)[MW], \forall t \in T$
- Vertical bidding curve  $C_{pr}(P(t))$  with point in  $P_{pr}(t)$  for every time step t for primary reserve market:  $P_{pr}(t)[MW], \forall t \in T$
- Two vertical bidding curves  $C_{sr,up}(P(t))$  with point in  $P_{sr,up}(t)$  and  $C_{sr,down}(P(t))$  with point in  $P_{sr,down}(t)$  for every time step t for secondary reserve market:  $P_{sr}(t)[MW], \forall t \in T$  and
- Vertical bidding curve  $C_{tr}(P(t))$  with point in  $P_{tr}(t)$  for every time step t for tertiary reserve market:  $P_{tr}(t)[MW], \forall t \in T$

According to assumption that GENCO Lokve-Bajer is price-*taker* electricity producer, it will be sufficient to create just vertical bidding curves since bids will be accepted with forecasted marginal clearing price (MCP) regardless of submitted quantity. In deterministic model that is presented in this paper, electricity price and also ancillary services prices are not forecasted. Instead, hourly prices for electricity only market and also ancillary services markets are taken from day ahead electricity market and day ahead ancillary services market on NYISO pool for day 09.6.2012.

### 9. CASE STUDY AND RESULTS

Prices of electricity (electric energy day-ahead – EE DA), the primary regulation, the secondary regulation, 10-minute spinning reserve and cold (the tertiary) reserve are taken from day-ahead auction market. Real-time electricity price (electric energy real-time – EE RT) is taken from real-time electricity market where hourly prices are equal to pondered average prices during that hour. All data is taken from NYISO electricity market and ancillary services markets on 09.06.2012 and are shown in figure 7.



Figure 7 Hourly prices taken from NYISO on 09.06.2012. for zone N.Y.C. (New York City).

Acquired results from optimization runs have shown that HPP Vinodol has by far largest role in HPS Lokve-Bajer and therefore has the largest effect on profit of HPS Lokve-Bajer as a whole. Profit, when HPP Vinodol participated only in the electricity market and only HPP Vinodol profit is maximized, is equal to 72 629 \$. If profit of HPS Lokve-Bayer is maximized then it reaches 76 580 \$. It is obvious that HPP Vinodol share of total profit is around 95% and consequently decided to optimize only HPP Vinodol profit has been made. This simplification has positive effects on calculation and optimization process. Case study data are presented in table 8 and table 9.

Reservoir	Križ	Lokve	Bajer	Lepenica	Potkoš
[m³/s]	2	8	6	2	2

Table VIII. Average natural inflows during day in reservoirs

Reservoir	Volume [hm <sup>3</sup> ]	HPP/PS	Discharge [m <sup>3</sup> /s]	Power [MW]
Križ	0.06	PS Križ	1.1	0.34
Lokve	34.8	PHPP Fužine	10/9*	4.6/4.8*
Bajer	1.32	HPP Vinodol	18.6	94.5
Lepenica	4.26	PHPP Lepenica	6.2/5.3*	1.14/1.25*
Potkoš	0.33	PS Lič	0.45	0.36

Table IX. Parameters of HPP/PS\* and reservoirs in HPS Lokve-Bajer

On figure 8 HPP Vinodol schedule while participating only in the electricity market is shown. If HPP Vinodol additionally participated in the primary regulation its daily profit is increased by 5,17% and is equal to 79430 \$. HPP Vinodol schedule in this new environment is shown in figure 9. Pee(t) represents HPP Vinodol part of capacity for electricity production and Preg(t) represents HPP Vinodol part of capacity intended for primary regulation. The probability that HPP Vinodol will participate in the primary regulation market is shown in table 10.

Table X. Probability that HPP Vinodol will participate in primary regulation

$p_{r,up}$	p <sub>r,down</sub>	$(1 - p_{r,up} - p_{r,down})$
0.40	0.35	0.25



Figure 8 HPP Vinodol schedule while participating only in electricity market



Figure 9 Available capacity and schedule of HPP Vinodol while participating in electricity market and primary regulation market

On figure 10 HPP Vinodol schedule while participating in electricity market and spinning reserve market is shown. If HPP Vinodol additionally participate in spinning reserve its daily profit is equal to 74731 \$.

On figure 11 HPP Vinodol schedule while participating in the electricity market, the primary regulation, the spinning reserve market and the cold reserve market is shown. If HPP Vinodol additionally participated in the primary regulation, the spinning reserve market and the cold reserve market its daily profit is equal to 81643 \$. Pee(t) represents HPP Vinodol part of the capacity for electricity production and Preg(t) represents HPP Vinodol part of the capacity intended for the primary regulation. Prot,up(t) represents HPP Vinodol part of the capacity for the secondary-up regulation. Prot,down(t) represents HPP Vinodol part of the capacity for secondary-down regulation. Pcold(t) represents HPP Vinodol part of the capacity for the tertiary regulation.



Figure 10 Available capacity and schedule of HPP Vinodol while participating in electricity market and spinning reserve market.



Figure 11 Available capacity and schedule of HPP Vinodol while participating in electricity market, primary regulation, spinning reserve market and cold reserve market.

#### 10. CONCLUSION

48

In this paper, it has been elaborated and shown that due to their high flexibility and robustness hydropower plants are capable of providing all sorts of ancillary services, from primary control to providing a cold reserve in a system. Assumptions that hydropower owners can increase expected profit by biding on several markets at the same time have been proven right. Results have shown by additional bidding on primary regulation market HPP Vinodol can increase expected profit by approximately 5% in comparison to the case where it only bids to the electricity market. If bids are also submitted to the spinning reserve and the cold reserve market this increase can reach 8%. This simultaneous bidding imposes a certain new risk to the owners of hydropower plants due to exposure to a wider range of market risks. But at the same time opportunities for an additional profit and to economic flexibility are increased. This paper shows that in near deterministic environment such as day-ahead markets it is possible to boost expected profit notably. It is of course almost impossible to utilize these opportunities without certain adequate support tool. One such tool is presented in this paper. Although rather simple due to computing limitations, presented model is efficiently found an optimum schedule of different services besides just bidding on the electricity market. Presented results are based on one specific case study of hydropower system Lokve-Bajer and therefore cannot be treated as a general estimate of the presented method. But at the same time, the model is flexible

enough to be adjusted to different specific locations to apply a similar analysis as presented in this paper.

### 11. **REFERENCES**

12.

- D. Zhang, P. B. Luh and Y. Zhang, "A Bundle Method for Hydrothermal Scheduling," *IEEE Transactions on Power Systems*, vol. 14, no. 4, pp. 1355-1361, 1999.
- [2] A. George, C. Reddy and A. Y. Sivaramakrishnan, "Multi-objective, short-term hydro thermal scheduling based on two novel search techniques," *International Journal of Engineering Science and Technology*, vol. 2, no. 12, pp. 7021-7034, 2010.
- [3] J. S. Dhillon and D. P. kothari, "Multi-Objective Short-Term Hydrothermal Scheduling Based on Heuristic Search Technique," *Asian Journal of Information Technology*, vol. 6, no. 4, pp. 447-454, 2007.
- [4] J. L. Zhang and K. Ponnambalam, "Hydro energy management optimization in a deregulated electricity market," *Optimization in Engineering*, vol. 7, no. 1, pp. 47-61, 2006.
- [5] A. Horsley and A. G. Wrobel, "Profit-maximizing operation and valuation of hydroelectric plant: A new solution to the Koopmans problem," *Journal of Economic Dynamics and Control*, vol. 31, no. 3, pp. 938-970, 2007.
- [6] P. Ilak, Istovremeno participiranje hidroelektrane na tržištu električne energije i tržištima pomoćnih usluga, Zagreb: Sveučilište u Zagrebu Fakultet elektrotehnike i računarstva, diplomski rad, 2012.
- [7] J. Garcia-Gonzalez, E. Parrilla and M. A., "Risk-averse profit-based optimal scheduling of a hydro-chain in the day-ahead electricity market," *European Journal of Operational Research*, vol. 181, no. 3, pp. 1354-1369, 2007.
- [8] J. Garcia-Gonzalez, E. Parrilla, A. Mateo and R. Moraga, "Building optimal generation bids of a hydro chain in the day-ahead electricity market under price uncertainty," in *International Conference on Probabilistic Methods* Applied to Power Systems 2006. PMAPS, Stockholm, 2006.
- [9] D. De Ladurantaye, M. Gendreau and J. Potvin, "Optimizing profits from hydroelectricty production," *Elsevier*, October 2007.

- [10] P. Coulibaly and F. Anctil, "Real-time short-term natural water inflows forecasting using recurrent natural networks," in Proc. Of IJCNN '99, International joint conference on neural networks, 1999.
- [11] D. Druce, "Insights from a history of seasonal inflow forecasting with conceptual hydrologic model," *Journal of Hydrology*, 2001.
- [12] A. Franco, E. Lobato and L. Rouco, "Optimization of the Spanish Market Sequence by a Price-Taker Generating Firm," in *IEEE Bologna Power Tech Conference*, 2003.
- [13] A. Baillo, M. Ventosa, M. Rivier and A. Ramos, "Strategic Bidding Uncertanty in a Competitive Electricity Market," in *Presented on the 6th International Conference on a Probabilistics Methods Applied to Power Systems*, Madeira, 2000.
- [14] A. Conejo, J. Arroyo, J. Contreras and F. Villamor, "Self-Scheduling of a Hydro Producer in a Pool-Based Electricity Market," *IEEE Transactions on Power* Systems, vol. 17, no. 4, 2002 2002.
- [15] HEP. [Online]. Available at www.hep.hr.
- [16] J. Nanda, A. Mangla and S. Suri., "Some new findings on automatic generation control of an interconnected hydrothermal system with conventional controllers," *IEEE Transaction on Energy Conversion*, vol. 21, no. 1, pp. 187-194, 2006.
- [17] K. Vrdoljak and P. N., "Primjena optimalnog kliznog režima upravljanja u sekundarnoj regulaciji frekvencije i djelatne snage razmjene regulacijskim hidroelektranama," *Automatika*, vol. 51, no. 1, pp. 3-18, 2010.
- [18] Y. H. Moon, H. S. Ryu, K. B. Song and M. C. Shin, "Extended integral control for loadfrequency control with the consideration of generation-rate constraints," *Journal of Electrical Power & Energy Systems*, vol. 24, no. 4, 2002.
- [19] H. Y. Yamin, "Review on methods of generation scheduling in electric power systems," *Science Direct*, 2003.
- [20] R. A. Ponrajah, J. Witherspoon and F. D. Galiana, "Systems to optimize conversion efficiencies at Ontario Hydro's hydroelectric plants," *IEEE Trans. Power Syst.*, vol. 14, pp. 1044-1050, 1998.
- [21] I. Rajšl, , P. Ilak, M. Delimar and S. Krajcar, "Dispach Method for Independetly owned Hydro Power Plants in the Same River Flow," *Energies*, 2012.

- [22] V. Gruić, "Pomoćne usluge u elektroenergetskom sustavu," Zagreb.
- [23] V. Gruić and N. Mandić, "Regulacija frekvencije u uvjetima otvorenog tržišta električne energije," *Energija*, vol. 51, no. 2, 2002.
- [24] S. J. Kazempour, M. Hosseinpur and M. P. Moghaddam, "Self-Scheduling of a Joint Hydro and Pumped-Storage Plants in Energy, Spinning Reserve and Regulation Markets," *IEEE Xplore*, 2010.
- [25] S. E. Fleten and E. Pettersen, , "Construction bidding curves for a price-taking retailer in the Norwegian market," *IEEE Trans. Power Syst.*, vol. 20, pp. 701-708, 2005.
- [26] P. Andrianesis, P. Biskas and G. Liberopoulos, "An overivew of Greece's wholesale electricty market with emphasis on ancillary services," *Electric Power System Research*, 2011.
- [27] E. L. Miguelez and I. E. Cortes, "An overview of ancillary services in Spain," *Science Direct*, 2008.
- [28] M. Mehmedović, M. Stojsavljević, D. Nemec, A. Černicki, B. Horvat and B. Premer, "Sustav sekundarne regulacije frekvencije i snage razmjene hrvatskog EES -Idejno rješenje".
- [29] "Current state of balance management in south east europe document for the 8th Athens forum," 2006.
- [30] S. S. Oren, "Auction design for ancillary reserve products," 2002.
- [31] D. Kirchen and G. Strbac, Fundamentals of Power System Economics, John Wiley & Sons, 2004.